Assignment #8: Rocket Sizing — ME 415

due 11/10/2021 before midnight via Learning Suite — 50 possible points

---

8.1 Rocket sizing analysis for the Saturn V rocket. This is a 3-stage liquid-fuel rocket. Although the real rocket has three stages, we will only analyze the first stage. The others stages would follow essentially the same procedure. Because this is an actual rocket, you can perform sanity checks along the way (see wikipedia for example). Our goal is to size the tanks and figure out how long and heavy the body (for the first stage) needs to be. Some key data is provided below.

The velocity change is a key spec that depends on the needs of the mission. The parameter ΔV_{ideal} is the velocity change from Tsiolkovsky's rocket equation ignoring all losses (gravity, drag, steering, etc.). This stage uses five Rocketdyne F-1 engines with RP-1 (rocket propellant-1) as the fuel and LOX (liquid oxygen) as the oxidizer. The Wikipedia page for the Saturn V only lists the sea level specific impulse, but we'll spend most of the stage away from sea level. The value listed below is simply an average of sea level and vacuum specific impulse.

mass of other stages (+ payload)	667,800 kg	wikipedia: Saturn V
rocket diameter	10.1 m	wikipedia: Saturn V
ideal velocity change ΔV_{ideal}	3600 m/s	computed based on mission
specific impulse I_{sp}	283 s	wikipedia: Rocketdyne F-1
oxidizer-to-fuel ratio	2.27	wikipedia: Rocketdyne F-1
one engine mass	8400 kg	wikipedia: Rocketdyne F-1
density of RP-1	810 kg/m^3	wikipedia: RP-1
density of LOX	1141 kg/m^3	wikipedia: LOX
tank thickness	5 mm	random internet citizen
rocket thickness	2 cm	scaled up from smaller rocket
density of aluminum	2810 kg/m^3	aluminum density variation

The notional design for this stage is shown in the figure at the top of this spec sheet. The fuel tanks and rocket body are made of aluminum. Note that a stacked configuration is used and that about 10 meters of extra length is required below the tanks for the engines. It is easiest to estimate the required tank size by assuming that the tanks are cylinders and then add on some length to account for the fact that the round end caps will need to be longer than a pure cylinder (and there will be some ullage volume). Based on the picture in the above spec sheet I estimated about a 3 m increase in length relative to a pure cylinder.

Because the structural mass depends on the propellant mass, and the propellant mass depends on the structural mass, an iterative process is required. If you're struggling to know if you're on the right track, because this is an actual rocket, you could start with the known propellant mass and work your way through the equations checking your numbers against the actual rocket, then fine tune your estimate of the propellant mass if needed.

Report the following. Be sure to clearly show your work and assumptions.

(a) The propellant mass.

(b) The required body length for this stage.

(c) The structural mass.

8.2 Trajectory of the first stage. This problem continues from the previous and you will need to use some of the results. In this case we are interested in the flight trajectory. The Saturn V does not fly at a straight angle during the first stage and so it would be difficult to provide an accurate estimate using the closed-form rocket equation. Instead, we need to use numerical integration. We will ignore drag in this analysis.

As mentioned the heading angle changes significantly throughout the flight. I fit a curve to postflight trajectory data and computed the heading angle as a function of time during stage 1. Note that $\theta = 0$ corresponds to vertical flight:

$$\theta = p_1 \arctan\left(p_2 t^{p_3}\right) \tag{1}$$

where

$$p_1 = 0.866, p_2 = 2.665 \times 10^{-5}, p_3 = 2.378 \tag{2}$$

We just need one additional parameter in addition to those already supplied.

thrust	35.1 MN	wikipedia: Saturn V

You could solve this using any ODE solver (e.g. `ode45` in matlab), though perhaps the easiest approach is just a forward Euler method. This means that you setup a time vector, and a starting point for V, m, z, x, and then execute a for loop. At iteration (i) you update those four values using data from the previous iteration $(i-1)$. For example, using the last ODE (and setting $\Delta t = t^{(i)} - t^{(i-1)}$ since it occurs for all four ODEs):

$$x^{(i)} = x^{(i-1)} + V^{(i-1)} \sin\theta^{(i-1)} \Delta t \tag{3}$$

Report the following. Be sure to clearly show your work and assumptions. The actually velocity and altitude at the end of stage 1, according to Apollo 11 data are 2400 m/s at 67 km altitude and 100 km forward distance.

(a) Plot the trajectory with forward distance on the x-axis and altitude on the y-axis.

(b) The final velocity, altitude, and forward distance.