Boundary Layers

Lecture 8

ME EN 412
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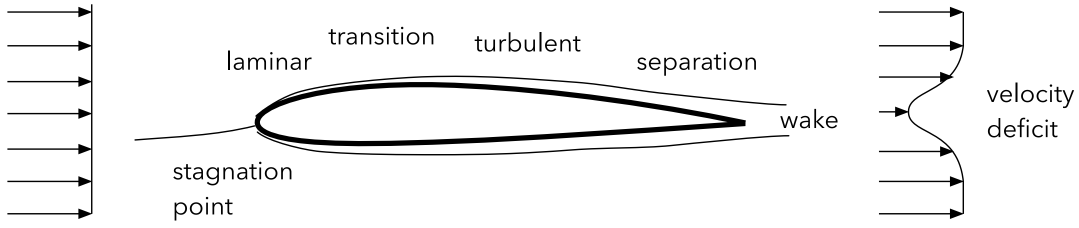
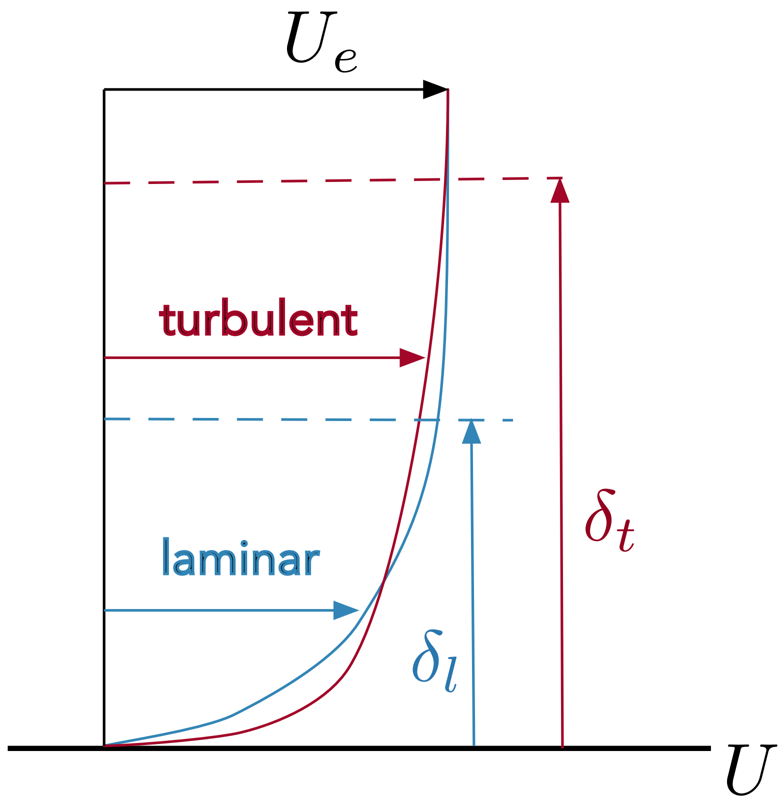
Outline

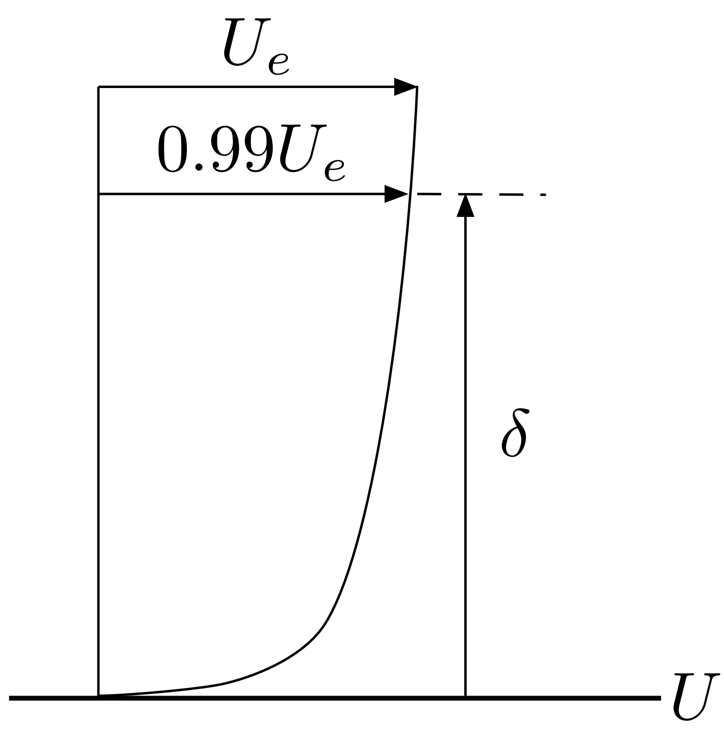
Boundary Layer Fundamentals

What is a boundary layer?

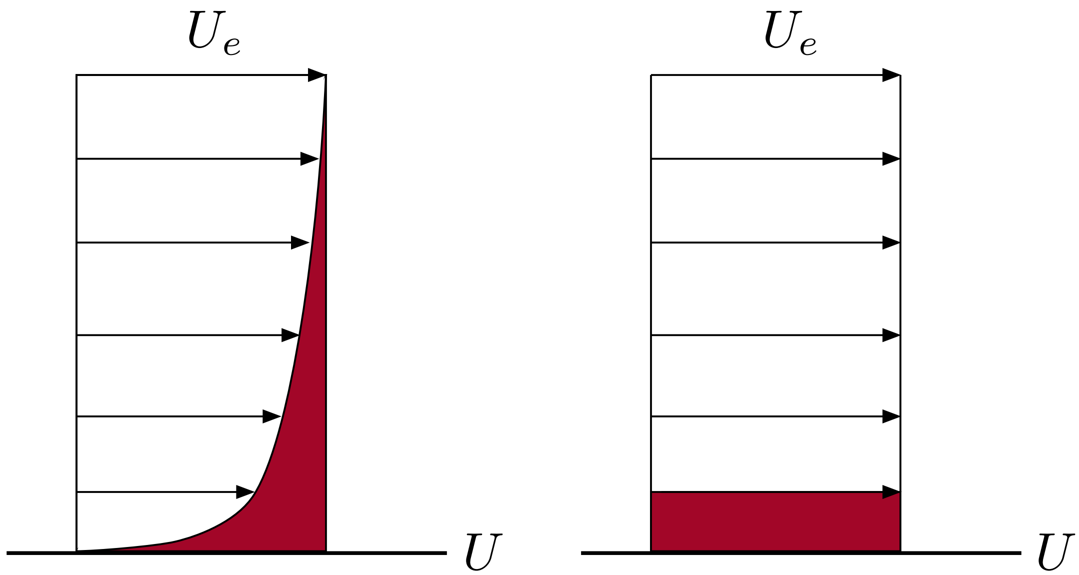




Boundary Layer Thickness

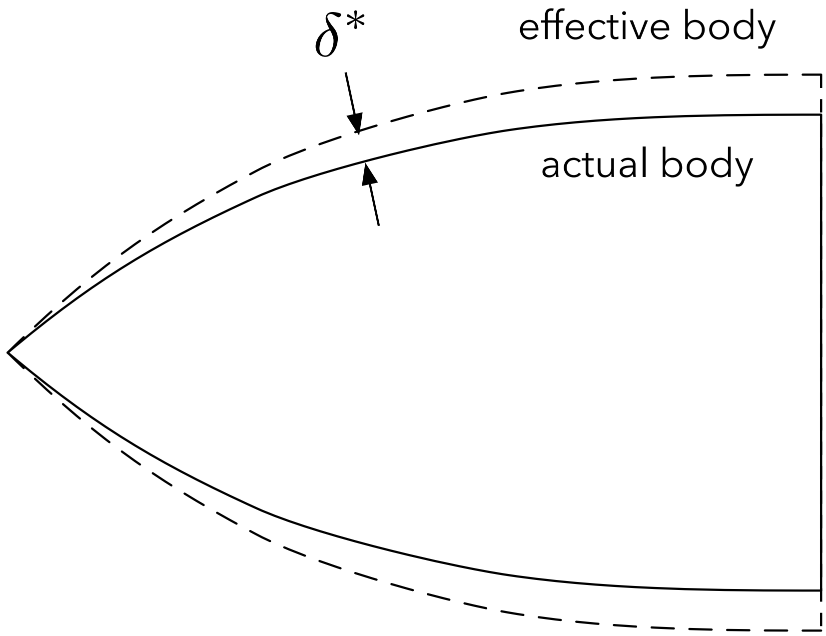


Displacement Thickness

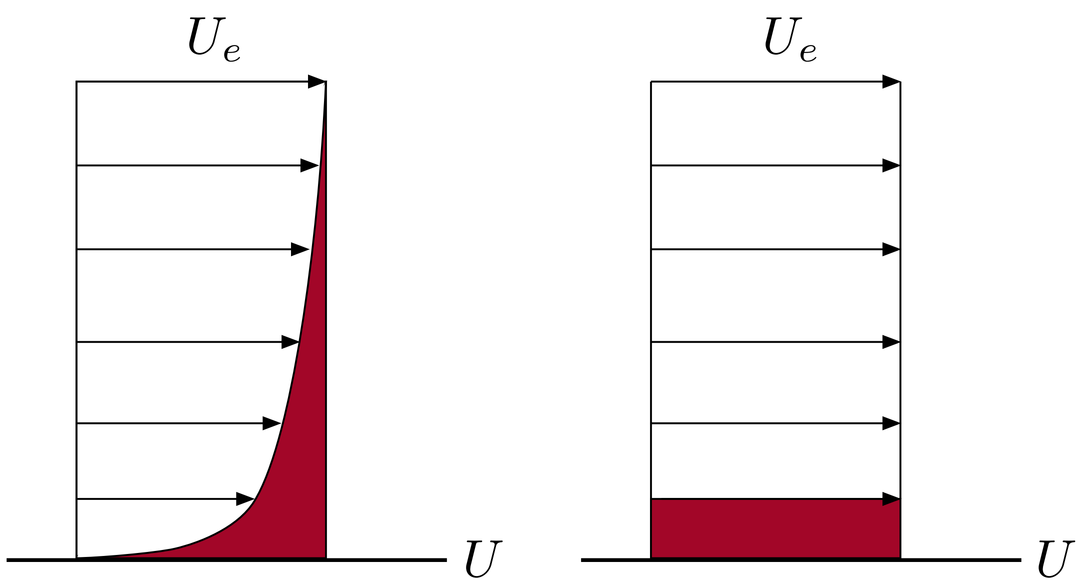


$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e V_e}\right) dy$$

A practical use of δ^*



Boundary Layer Momentum Thickness



$$\theta = \int_0^\infty \frac{\rho u}{\rho_e U_e} \left(1 - \frac{u}{U_e}\right) dy$$

Blasius Solution

Within a boundary layer, how are these related?

$$\frac{\partial}{\partial y} \quad ? \quad \frac{\partial}{\partial x}$$

and

$$u \quad ? \quad v$$

$$\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Analytic solution (flat plate, laminar flow)

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$\delta^* = \frac{1.72x}{\sqrt{Re_x}}$$

$$\theta = \frac{0.664x}{\sqrt{Re_x}}$$

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

$$c_f = \frac{\tau_w}{q_\infty}$$

$$c_f = \frac{0.664}{\sqrt{Re_x}}$$

$$c_{df} = \frac{1.328}{\sqrt{Re_L}}$$

empirical relationships for a flat plate

For turbulent flow there is no analytic solution

Schlichting:

$$\delta = \frac{0.37x}{Re_x^{0.2}} \qquad \delta^* = \frac{0.046x}{Re_x^{0.2}} \qquad \theta = \frac{0.036x}{Re_x^{0.2}}$$

$$c_f = \frac{0.0592}{Re_x^{0.2}}$$

$$c_{df} = \frac{0.455}{(log_{10} Re_L)^{2.58}} \text{ or } c_{df} = \frac{0.074}{Re_L^{0.2}}$$