Wind Turbines and Propellers

Lecture 31-32

ME EN 412
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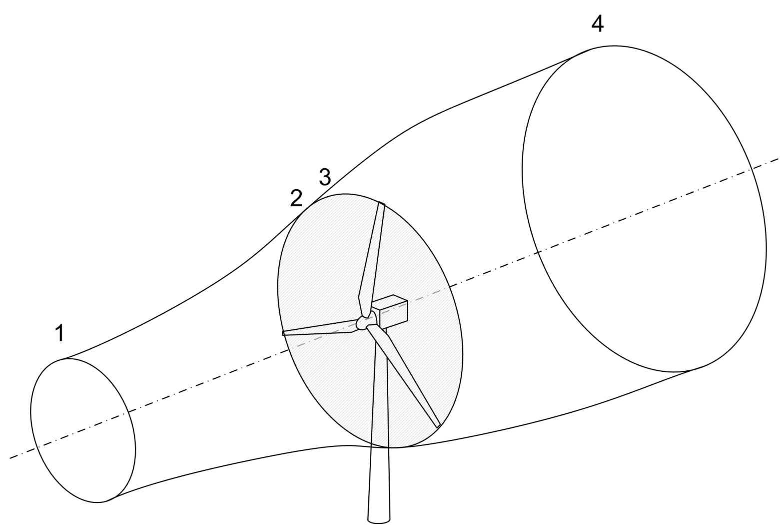
Outline

Momentum Theory

Blade Element Theory

Momentum Theory

Momentum Theory

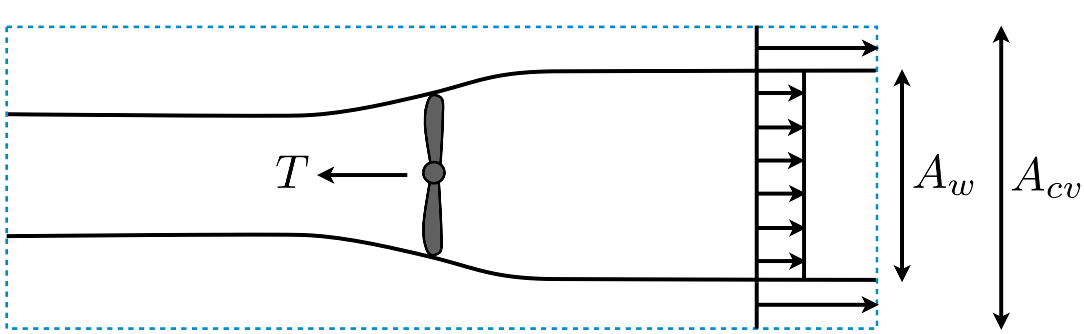


mass balance:

momentum balance:

combine:

It may not be obvious, that the pressure terms from the sides cancel (they do). We can come up with the same result more rigorously, with the below control volume.



Use a second control volume just across the disk (2-3)

Recall $V_2 = V_3 = V_d$

and $A_2 = A_3 = A_d$

momentum balance:

Control Volume 1:

$$T = \rho A_2 V_2 (V_1 - V_4)$$

Control Volume 2:

$$T = A_2 (P_2 - P_3)$$

Combine:

$$\rho V_2 (V_1 - V_4) = (P_2 - P_3)$$

We can find the pressure change from Bernoulli's

1-2:

3-4:

Combine:

Replace pressure in previous thrust expression:

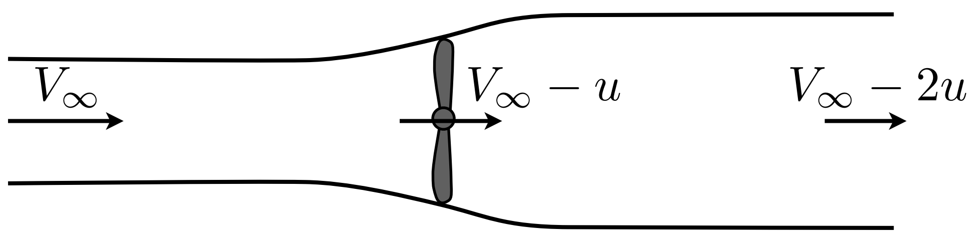
$$\rho V_d(V_1 - V_4) = (P_2 - P_3)$$

$$\rho V_d(V_1 - V_4) = \frac{1}{2}\rho(V_1^2 - V_4^2)$$

$$V_d(V_1 - V_4) = \frac{1}{2}(V_1 - V_4)(V_1 + V_4)$$

$$V_d = \frac{1}{2}(V_1 + V_4)$$

Thus, the velocity at the disk is half way between the upstream and downstream velocity.



Recalling the importance of nondimensional numbers:

$$
\begin{aligned}
V_d &= V_\infty - u \\
&= V_\infty \left(1 - \frac{u}{V_\infty}\right) \\
&= V_\infty \left(1 - a\right)
\end{aligned}
$$

Similarly, we know:

$$V_w = V_\infty(1 - 2a)$$

Thrust:

Thrust Coefficient:

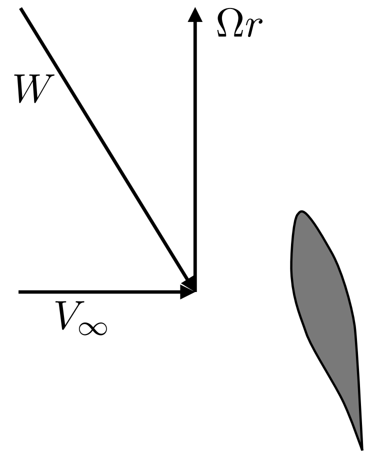
$$C_T = \frac{T}{\frac{1}{2}\rho V_\infty^2 A_d}$$

$$= 4a(1-a)$$

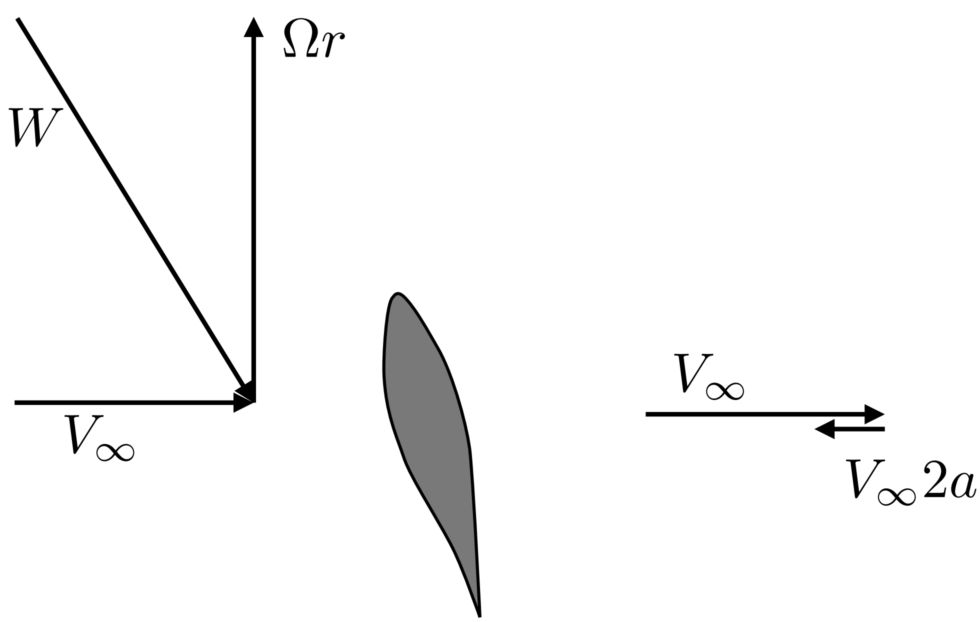
Angular Momentum Balance

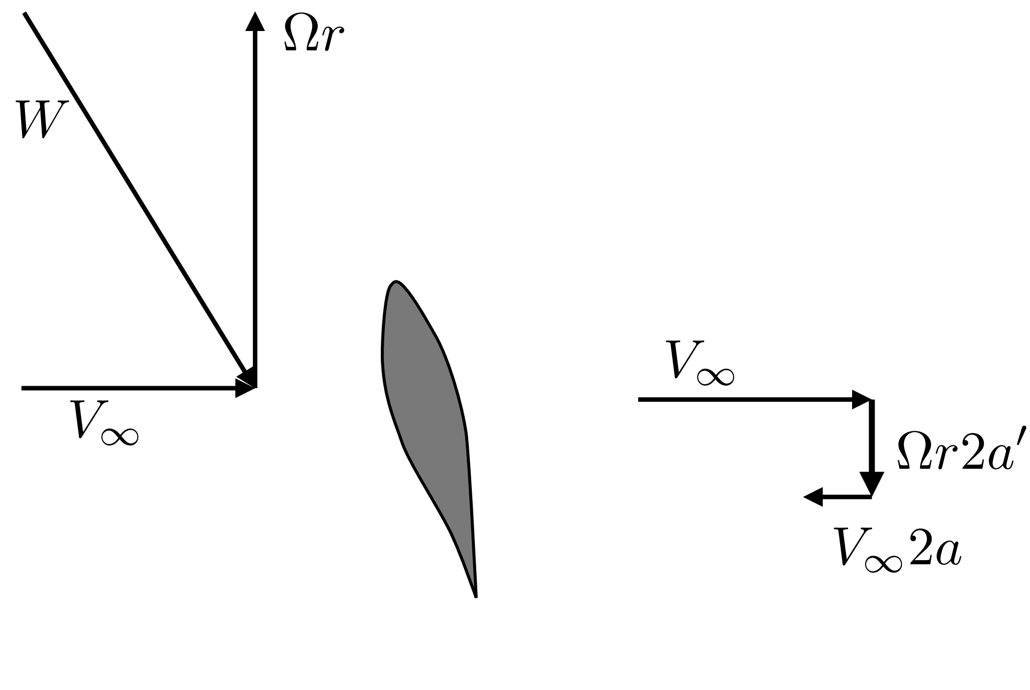
$$\int_S \left(\vec{r} \times \vec{V}\right) \dot{m} = \sum \vec{Q}$$

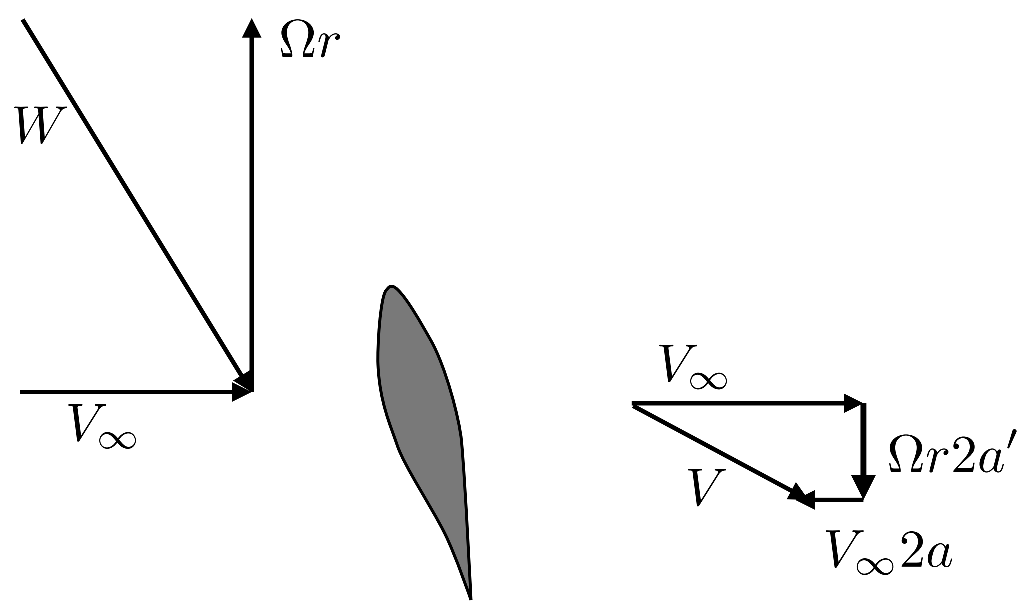
Velocity triangle*:

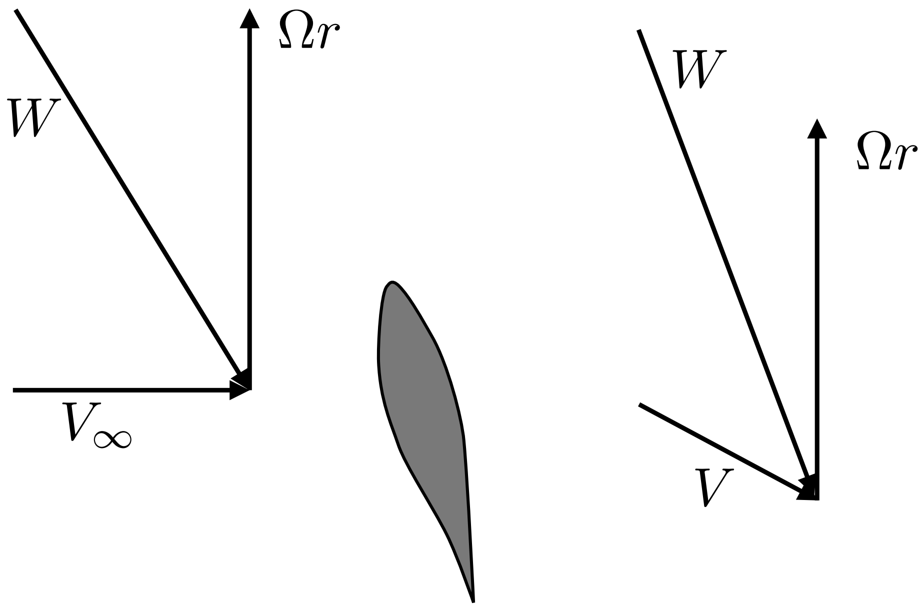


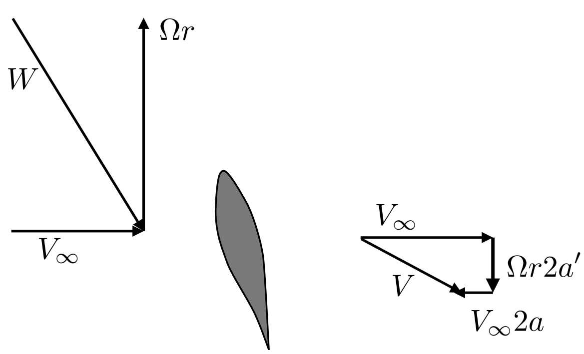
*This is an unconventional frame of reference and orientation for a wind turbine, but I use it because it will be more familiar to you as it matches the style of the book.




Ωr
W
V_∞
V_∞
$\Omega r 2a'$
$V_\infty 2a$


Ωr
W
V_∞
V_∞
$\Omega r 2a'$
V
$V_\infty 2a$





Apply angular momentum balance:

mass flow rate:

Torque:

Torque Coefficient:

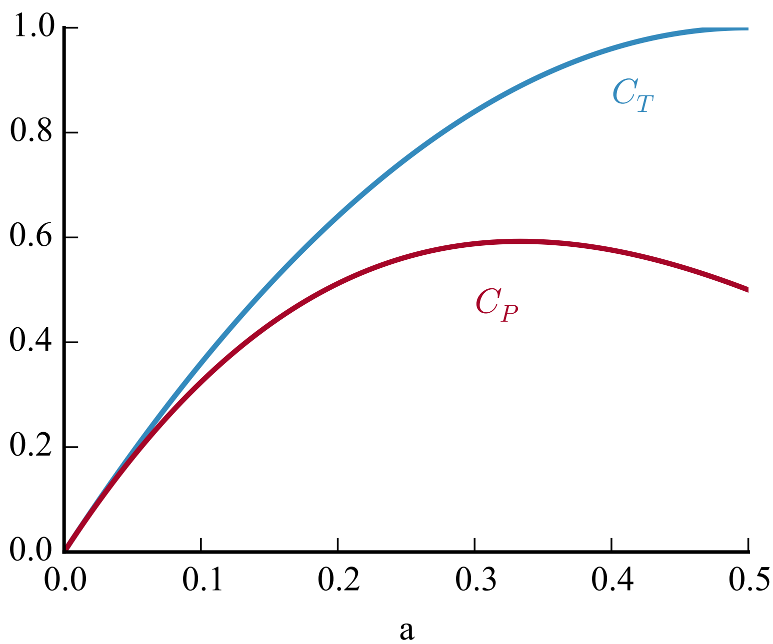
$$C_Q = \frac{Q}{\frac{1}{2}\rho V_\infty^2 A_d r}$$
$$= 4a'(1-a)\lambda_r$$

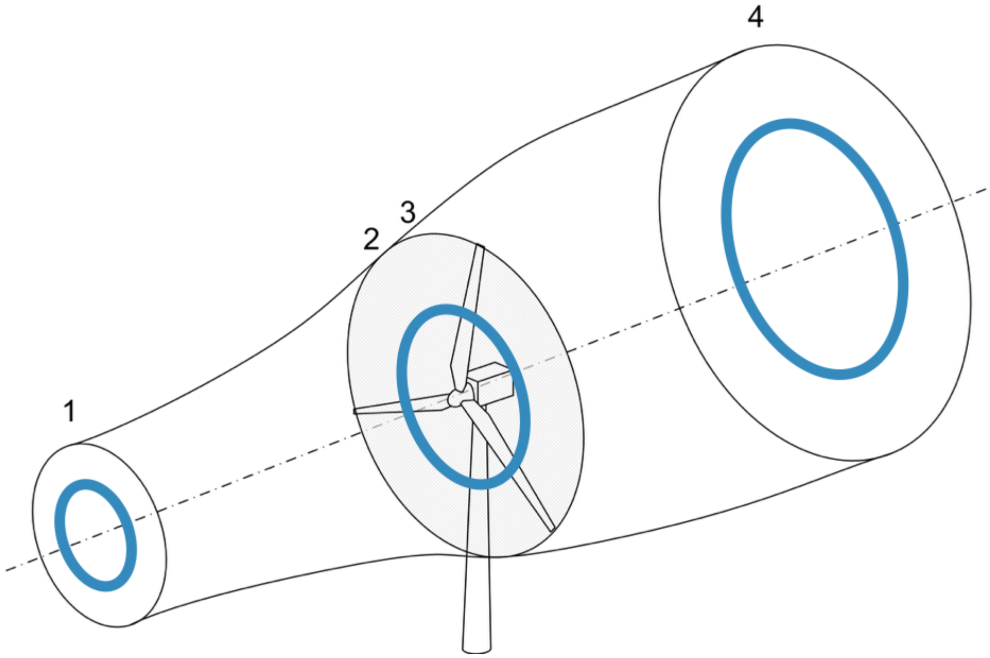
where $\lambda_r = (\Omega r / U_\infty)$ is called the tip speed ratio

Momentum Theory:

$$C_T = 4a(1-a)$$

$$C_Q = 4a'(1-a)\lambda_r$$

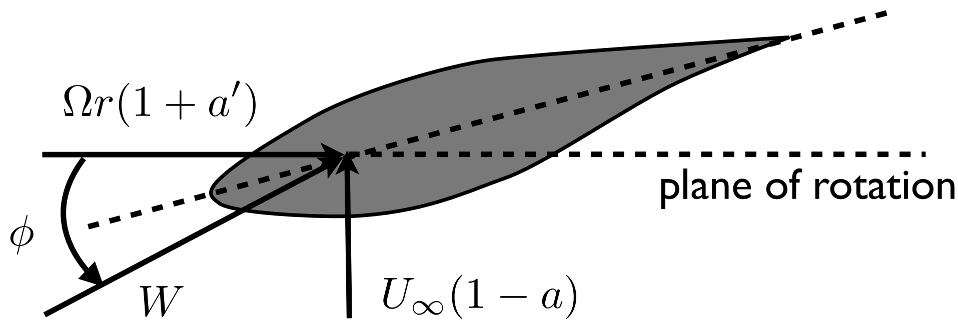




Blade Element Theory

Blade Element Theory

Velocity triangle at the blade, in the reference frame of the blade:



angle of attack:

lift and drag coefficients:

normal and tangential force coefficients:

elemental thrust:

elemental torque:

thrust coefficient:

torque coefficient:

find W from velocity triangle:

Local solidity:

$$\sigma' = \frac{Bc}{2\pi r}$$

Local tip-speed ratio:

$$\lambda_r = \frac{\Omega r}{U_\infty}$$

Momentum Theory:

$$C_T = \left(\frac{1-a}{\sin\phi}\right)^2 c_n \sigma'$$

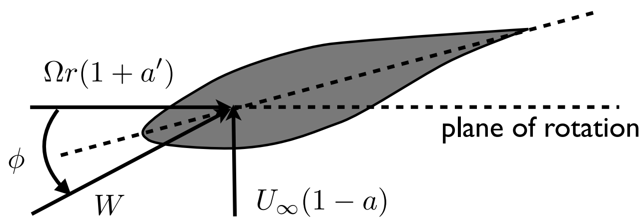
$$C_Q = \left(\frac{1+a'}{\cos\phi}\right)^2 c_t \sigma' \lambda_r^2$$

If we equate these two theories we can solve for the unknown induction factors

$$a = \frac{\sigma' c_n}{4\sin^2\phi + \sigma' c_n}$$

$$a' = \frac{\sigma' c_t}{4\sin\phi\cos\phi - \sigma' c_t}$$

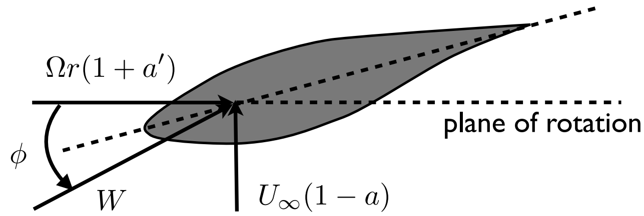
Problem: ϕ depends on a and a'



Conventional Solution: Fixed point iteration, or a 2D root solve.

New Problem: does not always converge.

Better Solution*: Change variables to ϕ and W:



$$\mathcal{R}(\phi) = \frac{\sin\phi}{1 - a(\phi)} - \frac{\cos\phi}{\lambda_r(1 + a'(\phi))} = 0$$

* Ning, A., "A Simple Solution Method for the Blade Element Momentum Equations with Guaranteed Convergence," Wind Energy, Vol. 17, No. 9, Sep. 2014, pp. 1327–1345, doi:10.1002/we.1636.

Algorithm	Avg. Function Calls	Failure Rate (%)
Fixed-Point	31.8	12.6
Newton	79.0	5.8
Steffensen	16.4	16.3
Powell Hybrid	72.3	16.2
Levenberg-Marquardt	92.3	8.8
New Method	**11.3**	**0.0**