Converging Diverging Nozzles

Lecture 25

ME EN 412
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Outline

Mass Conservation

Changing Pressure Ratio Across a Nozzle

Example

Mass Conservation

Mass Conservation

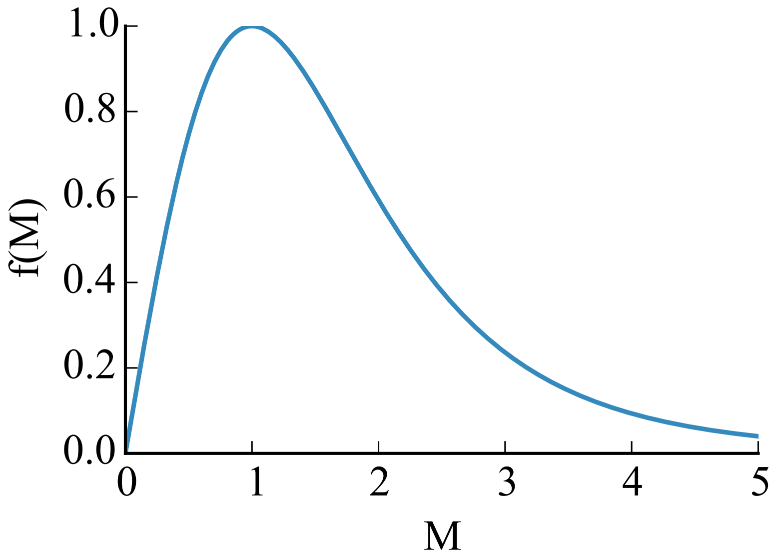
$$\frac{A^*}{A} = M \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$$\left(\frac{P_T A f(M)}{\sqrt{T_0}}\right)_1 = \left(\frac{P_T A f(M)}{\sqrt{T_0}}\right)_2$$

This is the most imporant equation for nozzle analysis.

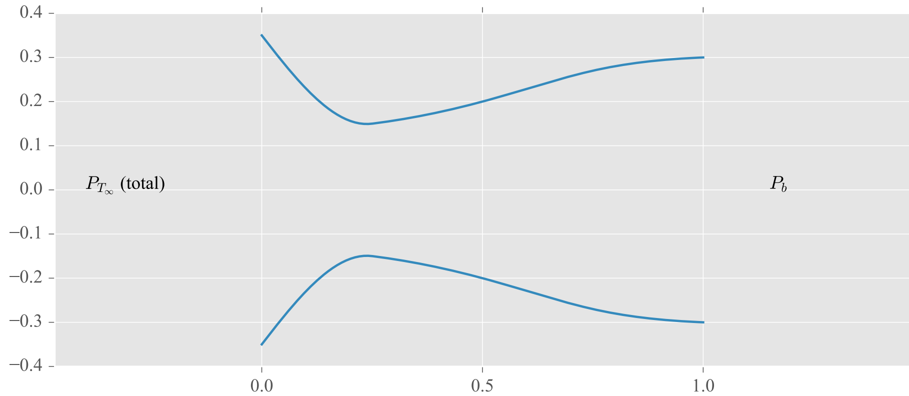
Simplifications

Here is what $f(M)$ looks like for air:



Changing Pressure Ratio Across a Nozzle

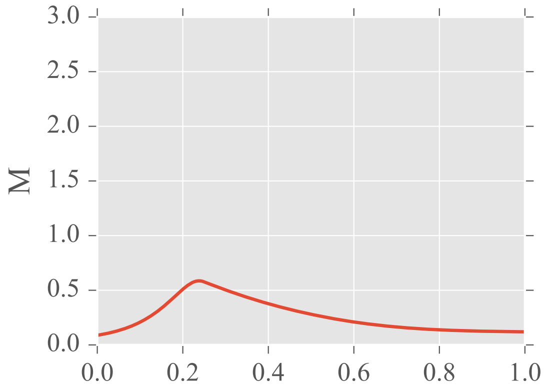
Changing Pressure Ratio Across a Nozzle

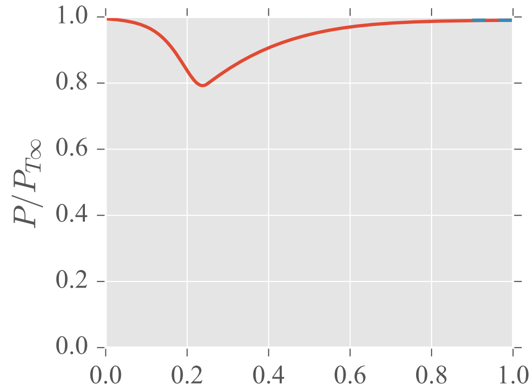


Important parameter is: $\frac{P_b}{P_{T\infty}}$

What happens when $\frac{P_b}{P_{T\infty}} = 1$?

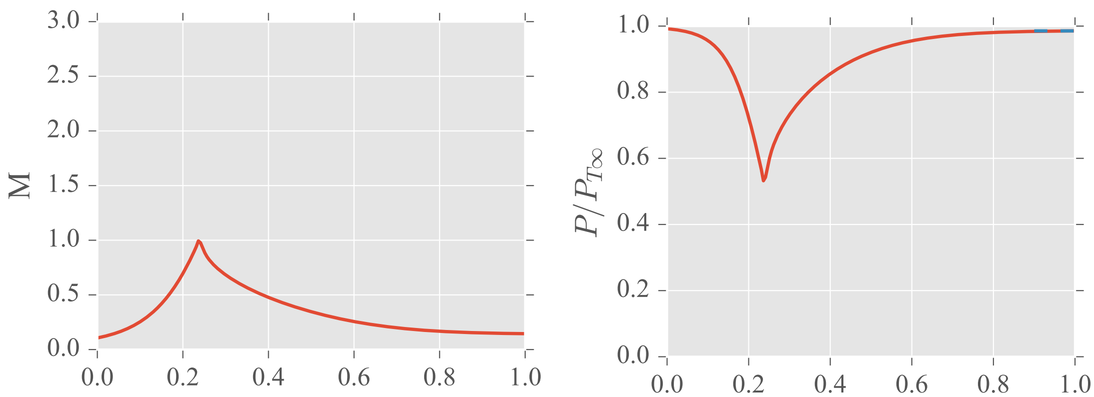
Now what happens if we slowly decrease $\frac{P_b}{P_{T\infty}}$ below 1?





How can we compute the Mach number at a given location?

Continue to decrease the back pressure up until the throat just barely becomes sonic.



The nozzle is now choked.

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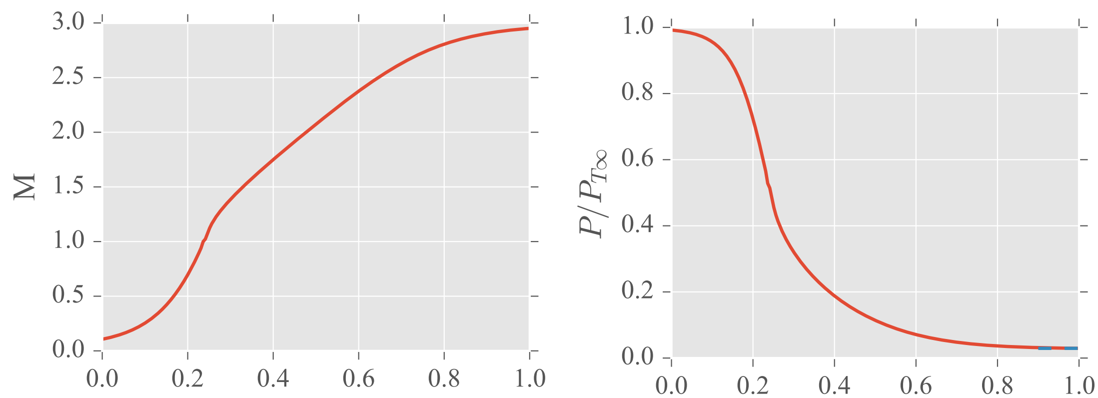
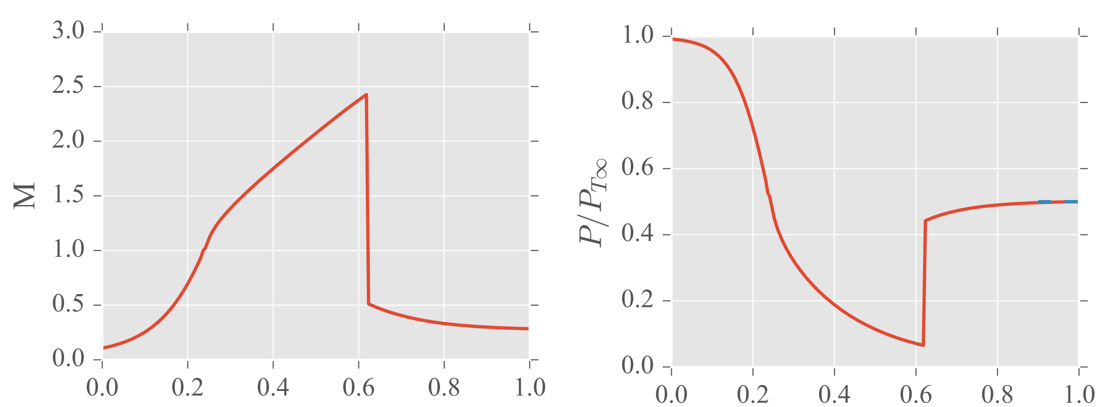
Anderson, Modern Compressible Flow

Now what happens as we decrease the back pressure a little more?

Recall that the supersonic isentropic solutions looks like this:

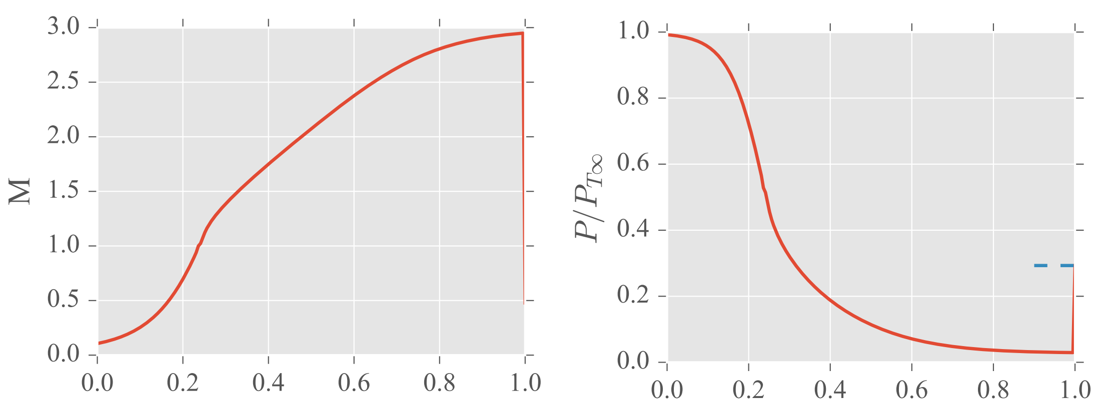


Shock wave



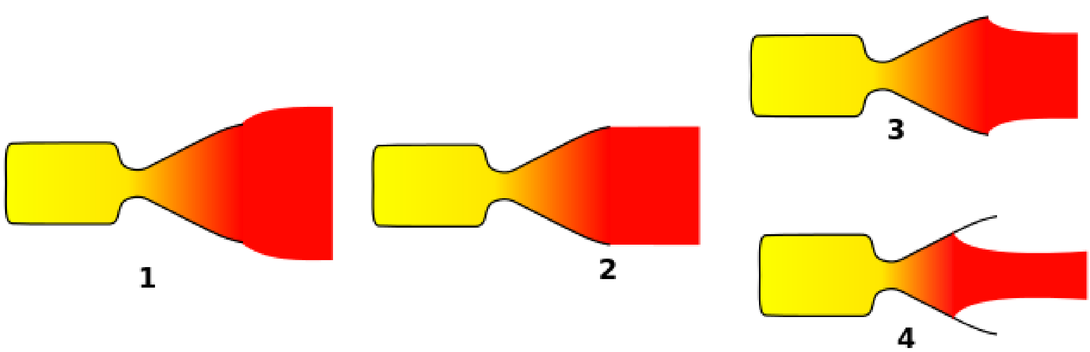
Assume we know the Mach number before the shock and its location (we can compute this), how can we compute Mach number elsewhere?

What happens we we continue to decrease the back pressure?



As we decrease the back pressure further the exit is supersonic and the exit pressure and back pressure may no longer match. There are three cases:

- overexpanded
- ideally expanded
- underexpanded



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1: under-expanded
2: ideally expanded
3: over-expanded
4: highly over-expanded

Jupyter notebook

Example

Example

Normal shock in nozzle.