Balance Laws 1

Lecture 2

ME EN 412
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Outline

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Fundamental Principles

Mass Balance

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Practice Problems

Prob 7.37

A sphere of diameter d falls slowly in a highly viscous fluid. What parameters might be important?

Only a limited number of experiments can be performed. Can we build a predictive model for the settling velocity?

From one experimental run:

- $V = 0.42$ ft/s
- $d =$ 0.1 in
- $\mu = 0.03$ lb-s /ft^2
- $\Delta\gamma = 10$ lb/ft^3

Fundamental Principles

- Mass is conserved.
- $F = ma$ (Newton's second law) and its angular counterpart.
- Energy is conserved (first law of thermodynamics).
- Entropy will always increase over time (second law of thermodynamics). It can be produced but not destroyed.

All of these concepts can be expressed in terms of balance laws:

$$\text{rate of accumulation} = \text{rate of inflow} \\ - \text{rate of outflow} + \text{rate of production}$$

Mass Balance

Derive mass balance.

$$\frac{\partial}{\partial t}\int_{\forall}\rho d\forall + \int_{S}\rho\vec{W}\cdot d\vec{A} = 0$$

W is the relative velocity

Total velocity:

$$\vec{V} = \vec{W} + \vec{V}_V$$

Momentum Balance

$$\begin{aligned} &\text{rate of momentum accumulation} + \text{rate of outflow} \\ &-\text{rate of inflow} = \text{rate of production} \end{aligned}$$

Newton's 2nd Law

$$\Sigma\vec{F} = \frac{d(m\vec{V})}{dt}$$

$$\frac{\partial}{\partial t}\int_{\forall}\rho\vec{V}d\forall+\int_{S}\rho\vec{V}(\vec{W}\cdot d\vec{A})=\Sigma\vec{F}$$

Any external forces can be applied, but the most common are the fluid pressure forces, fluid viscous forces, and gravitational forces.

Pressure:

$$\Sigma\vec{F}_p=-\int_{S}pd\vec{A}$$

Viscous shear stress:

$$\Sigma\vec{F}_v=\int_{S}\overleftrightarrow{\tau}\cdot d\vec{A}$$

$$\frac{\partial}{\partial t}\int_{\forall}\rho\vec{V}d\forall + \int_{S}\rho\vec{V}(\vec{W}\cdot d\vec{A}) =$$

$$-\int_{S}pd\vec{A} + \int_{S}\overset{\leftrightarrow}{\tau}\cdot d\vec{A} + \sum\vec{F}_{other}$$