CFD Theory

Lecture 19

ME EN 412
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Outline

Discretization Methods

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- Finite Difference
- Finite Volume
- Finite Element

Finite Difference Method

Finite Difference

Forward/Backwards difference

Central difference

Second derivative

Non-equally spaced mesh

Explicit and Implicit

Explicit vs Implicit

Consider a model equation like

$$\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0$$

(e.g., continuity where $F = \rho u$ and $G = \rho v$)

Explicit: Vectorizable, easier to implement, but requires smaller step sizes for stability.

Implicit: Can take larger step sizes, but memory intensive.

Example

$$k_1 \frac{\partial u}{\partial y} + k_2 \frac{\partial^2 u}{\partial y^2} = k_3$$

Matrix form for a column (e.g., constant i):

$$\begin{bmatrix} a_1 & b_1 & c_1 & & & & & & \\ & \ddots & & & & & & & \\ & & a_{j-1} & b_{j-1} & c_{j-1} & & & & \\ & & & a_j & b_j & c_j & & & \\ & & & & a_{j+1} & b_{j+1} & c_{j+1} & & \\ & & & & & \ddots & & & \\ & & & & & & a_N & b_N & c_N \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_{j-1} \\ u_j \\ u_{j+1} \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_{j-1} \\ d_j \\ d_{j+1} \\ \vdots \\ d_N \end{bmatrix}$$

Conservation Form

Conservation Form

Euler equations in conservation form

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} + Q = 0$$

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ e \end{bmatrix}$$

$$F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (e+p)u \end{bmatrix}$$

$$G = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ (e+p)v \end{bmatrix}$$

$$H = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ (e+p)w \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 \\ \rho f_x \\ \rho f_y \\ \rho f_z \\ p\dot{q} + \rho(uf_x + vf_y + wf_z) \end{bmatrix}$$