

CFD Turbulence

Lecture 18



ME EN 412
Andrew Ning
aning@byu.edu

Outline

Turbulent Length Scales

Direct Numerical Simulation (DNS)

Reynolds Averaged Navier Stokes

Large Eddy Simulation

Turbulent Length Scales

Turbulent Length Scales



$Re = 2,500$



$Re = 10,000$

Direct Numerical Simulation (DNS)

Direct Numerical Simulation (DNS)

In a DNS simulation, the Navier-Stokes equations are solved without any turbulence modeling, meaning that **all** spatial and temporal scales are resolved.

Example: [YouTube](#)

Reynolds Averaged Navier Stokes

Reynolds Averaged Navier Stokes

Copyrighted image (can only be shown in class)

$$\langle X \rangle = E[X] = \int_{-\infty}^{\infty} x f(x) dx$$

where $f(x)$ is a probability density function.

for a discrete variable we would express this as

$$\langle X \rangle = \sum_{i=1}^{\infty} x_i p_i$$

Reynolds decomposition

$$u = \langle u \rangle + u'$$

Example: 2D Incompressible Navier Stokes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

With time averaging:

$$\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} = 0$$

$$\begin{aligned} \rho \left(\langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} \right) &= -\frac{\partial \langle p \rangle}{\partial x} \\ &+ \frac{\partial}{\partial x} \left(\mu \frac{\partial \langle u \rangle}{\partial x} - \rho \langle u'^2 \rangle \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial \langle u \rangle}{\partial y} - \rho \langle u'v' \rangle \right) \end{aligned}$$

and similarly, for the y-equation

Reynolds stresses

Note the new terms of the form

$$-\rho V_i V_j$$

Large Eddy Simulation

Large Eddy Simulation

Example: [YouTube](#)