Assignment #4: Boundary Layers — ME 412

due 9/28/2016 before midnight via Learning Suite — 50 possible points

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4.1 Munson 9.20

4.2 Munson 9.51. For simplicity, assume laminar flow across the entire blade even though the flow near the tip is almost certainly turbulent.

4.3 Open-ended:

(a) For a car, estimate how far from the front edge of the hood before the boundary layer becomes turbulent.

(b) What is the thickness of the boundary layer at this location.

(c) For a Boeing 747, estimate the boundary layer thickness at the aft end of the fuselage.

4.4 Computing: Use Head's method and the integral boundary layer equation to numerically compute the development of a turbulent boundary layer over on a flat plate. Details on Head's method are below. Assume a Reynolds number based on plate length of 1 million ($Re_L = 1e6$). You will plot four boundary layer quantities as a function of position on the plate. **Each quantity should be on a separate plot.** For each quantity, compare your results to the Schlichting power law estimates for a turbulent boundary layer (in the lecture handout: "Boundary Layers"), **and** to the Blasius analytic solutions for a laminar boundary layer.

(a) Plot the displacement thickness δ^* along the plate

(b) Plot the momentum thickness, θ along the plate

(c) Plot the shape factor H along the plate

(d) Plot the local skin friction coefficient c_f along the plate

(e) Integrate c_f to compute a skin friction drag coefficient c_{df}.

To reiterate, you should have four plots and each plot should have three curves. You should also have three values for c_{df} (integral boundary layer, Schlichting, and Blasius).

Brief overview of Head's method:

Recall the integral boundary layer momentum equation discussed in class:

$$\frac{d\theta}{dx} + \frac{\theta}{U_e}(H+2)\frac{dU_e}{dx} = \frac{1}{2}c_f \tag{1}$$

This equation has three unknowns: θ, H, and c_f. To close the relationship Head used an empirically-derived relationship for skin friction of a turbulent boundary layer from Ludwieg-Tillman [1], and the concept of entrainment velocity paired with experimental data from Cebeci and Bradshaw [2] to create a relationship between H and θ. We will omit the details here, but the result is two linear ODEs, one for θ and one for an intermediate variable H_1:

$$\begin{aligned} \frac{d\theta}{dx} &= \frac{1}{2}c_f - \frac{\theta}{U_e}(H+2)\frac{dU_e}{dx} \\ \frac{dH_1}{dx} &= \frac{0.0306(H_1-3)^{-0.6169}}{\theta} - \frac{H_1}{U_e}\frac{dU_e}{dx} - \frac{H_1}{\theta}\left(\frac{1}{2}c_f - \frac{\theta}{U_e}(H+2)\frac{dU_e}{dx}\right) \end{aligned} \tag{2}$$

The shape factor, H, is given as a function of H_1:

$$\begin{aligned} H &= 1.1 + 0.86(H_1 - 3.3)^{-0.777} && \text{for } H_1 \geq 5.3 \\ &= 0.6778 + 1.1538(H_1 - 3.3)^{-0.326} && \text{for } 3.427 < H_1 < 5.3 \\ &= 3 && \text{for } H_1 \leq 3.427 \end{aligned} \tag{3}$$

and the empirical formula for c_f is given as a function of H and θ:

$$c_f = 0.246 \times 10^{-0.678H} Re_\theta^{-0.268} \tag{4}$$

We now have four equations with four unknowns (θ, H, H_1, c_f) that we can solve.

Hint: Recall that our case is a flat plate, so there is no pressure gradient and thus the edge velocity is constant ($\frac{dU_e}{dx} = 0$). This simplifies the above ODEs.

I have provided a few Matlab scripts for you:

- head.m (fully complete). Function that computes H and c_f as a function of θ and H_1 using the above formulas.
- headode.m (not complete). Function that takes in y = $[\theta,\ H_1]$, and should compute $dy/dx = [d\theta/dx, dH_1/dx]$ using the above formulas.
- flatplate.m (not complete). A script to solve the pair of ordinary differential equations using *ode45* (If you are using Python you would use *scipy.integrate.ode*). This is also where you will want to make comparisons to the Blasius and Schlichting relationships. The Matlab documentation has good examples of how to use ode45 (our case uses x whereas the Matlab examples use t).

4.5 CFD: Complete the tutorial Incompressible Flow/Steady Flow: Laminar and Turbulent in an S-Bend. Solve the external flow simulation you have setup the past couple weeks using an incompressible RANS solver. Create some visualizations of quantities of interest (e.g., pressure, velocity, etc.). Discuss the results and any deficiencies in your model. Review again the guidelines in the STAR-CCM+ User Guide: Best Practices > Guidelines for Aerodynamics Calculations > Incompressible External Aerodynamics: Steady State RANS Approach.

References

[1] Ludwieg, H. and Tillmann, W., *Investigations of the Wall-Shearing Stress in Turbulent Boundary Layers*, National Advisory Committee for Aeronautics Washington, 1950.

[2] Cebeci, T. and Bradshaw, P., *Momentum Transfer in Boundary Layers*, McGraw Hill, 1977.