

Assignment #3: Potential Flow

due 9/21/2016 before midnight via Learning Suite

ME 412
50 possible points

Project: Start learning a little about your proposed topic. Suggested avenues include reading textbooks and journal articles, watching videos and animations, talking to people with experience in the area, etc.

3.1 6.80 from Munson

3.2 By combining uniform flow and a doublet you can simulate incompressible, irrotational flow over a cylinder. Complete the following tasks for a cylinder with unit radius:

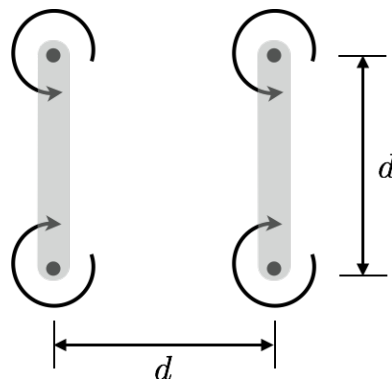
- (a) Derive an expression for the pressure coefficient and create a contour plot of the pressure coefficient for the flow around the **outside** of the cylinder. Use a domain 4 radii wide and 4 radii high.
- (b) Plot the pressure coefficient along the cylinder surface as you move from the front of the cylinder to the back.
- (c) Add a vortex to the center of the cylinder. Adjust the circulation strength and observe the effect on the streamlines. Discuss what you find and include a plot of the streamlines for one case.

3.3 Vortex rings are pretty neat. Read this [wikipedia article](#) to get an idea of what they are. They can propel themselves quite far when moving through a quiescent fluid. See some fun videos of vortex rings formed by [a volcano](#), [dolphins](#), and [a plate in a pool](#). They can also be dangerous, for example with helicopters that [descend too quickly](#) (skip to 1:15).

If you're really careful, and put one vortex ring right behind another you can have leapfrogging vortices! Check out this video of a [real-life visualization](#) of the phenomenon, as well as a [2D simulation](#), and a neat [3D simulation](#).

We are going to simulate the leapfrogging vortices in 2D. If you picture a toroidal vortex intersecting a 2D plane you will have two vortices (like the left pair shown below). From our understanding of vortices we can see how it is self-propelling. Consider the left pair of vortices (e.g., one ring vortex). The top vortex induces a velocity on the bottom vortex towards the right. Simultaneously, the top vortex induces a velocity on the top vortex to the right. Thus, it pushes itself forward. With two vortices next to each other, things become even more interesting.

In this example, make all vortices of equal strength (Γ). Each vortex is free to move, and would move with the local fluid velocity. Plot the trajectory of each vortex. You need to use a small enough time step to ensure stability. You may need to try smaller time steps to make sure that your simulation is accurate. Turn in a copy of your code as well.



Hints: In vector form, the induced velocity from a vortex is:

$$\begin{aligned}\vec{V} &= \frac{\vec{\Gamma} \times \hat{r}}{2\pi r} \\ &= \frac{\vec{\Gamma} \times \vec{r}}{2\pi r^2}\end{aligned}\tag{1}$$

Bonus (not required, not extra credit, just for fun): If you just look at trajectories it is hard to tell that they are leapfrogging. See if you can create an animation so that you can actually watch the movement in time. You might also like to draw a line between the vortex pairs to better visualize them as a connected ring.

- 3.4 Open-ended:** Point vortices provide good predictions of induced velocities for a lot of physical phenomena, but only when viewing velocities away from its center. The problem with the point vortices is that they are poor representations of reality near the center. For example the tangential velocity induced by a point vortex is

$$V_{\theta} = \frac{\Gamma}{2\pi r}$$

At $r = 0$ this model would predict infinite velocity! In reality, viscous forces are very important near the core of a vortex. One simple model treats the core of a vortex as rotating like a solid body (e.g, speed varies linearly with radius). This is called a [Rankine Vortex](#). This model works quite well in modeling hurricanes. (While an improved model, it is still overly simplistic for some applications, like for the vortices behind an aircraft—see for example Fig. 4 of [this paper](#)).

For Hurricane Katrina, plot the tangential velocity (in mph) as a function of radius from its center (in miles). Also plot pressure (in atmospheres) vs radius. Is the “size” (e.g., radial extent) of your vortex reasonable?

- 3.5 CFD:** Complete the “Mesh/Region Based Meshing Imported Control Valve” tutorial. Build an appropriate mesh for the external flow domain you created last week. Turn in several plots of your mesh (e.g., close in the boundary layer region, far-field view, etc.). Discuss your meshing strategy and size parameters that you used.

Meshing is one of the hardest aspects of CFD, and is a skill that takes a lot of practice. A good resource is found in the STAR-CCM+ User Guide: Best Practices > Guidelines for Aerodynamics Calculations > Incompressible External Aerodynamics: Steady State RANS Approach. Also, [this presentation](#) contains a good introduction to grid types and some things you should look for in a quality mesh.