

Fatigue Overview

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There are four scenarios we have discussed for analyzing fatigue:

1. Fully reversed simple loading (i.e., mean zero)
2. Fluctuating simple loading
3. Combined simple loading
4. Complex loading

The first three are what we will emphasize in this class, although there is a short homework problem on Case 4 to give you some exposure. Let's discuss the analysis steps for each of these cases.

1. Fully Reversed Simple Loading

- (a) Gather material data. This includes the ultimate strength S_{ut} , the yield strength S_y , and if available from test data the unmodified endurance limit S'_e . If the latter is not available from test data, and you are using steel, you can use the formula from (6-8) in the book. Recall that this predicts that the $S'_e = 0.5S_{ut}$ until some maximum threshold after which S'_e is a constant.
- (b) Compute the endurance limit after applying all the Marin factors

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \quad (1)$$

- (c) Compute the fatigue stress-concentration factors (if necessary). First find the static stress concentration factors from Table A-15 (K_t or K_{ts}). Then you need to obtain the notch sensitivity, either from the charts or the equations (Figs. 6-20 and 6-21, Eqns. 6-34 and 6-35). The notch sensitivity just defines a ratio between the static stress concentration factors and the fatigue stress concentration factors. The notch sensitivity is a function of the material, and geometry. You can compute the fatigue stress concentration factor K_f as

$$K_f = 1 + q(K_t - 1) \quad (2)$$

and similarly for K_{fs} (for shear stresses):

$$K_{fs} = 1 + q_s(K_{ts} - 1) \quad (3)$$

- (d) Compute the magnitude of the nominal stress reversal and apply the stress concentration factors.

$$\sigma_{rev} = \sigma_0 K_{fs} \quad (4)$$

- (e) Check for infinite life (i.e., is $\sigma_{rev} \leq S_e$).
- (f) If not infinite life, determine the corresponding number of cycles for a given safety factor, or determine the safety factor for a given number of cycles. First, the constants for the finite-life region are needed (a and b in $S_f = aS_{ut}^b$). These could be determined from experimental data

using linear regression in log-log space. Or, in this class, we can use the simplified formulas in the book that fit between two points:

$$a = \frac{(fS_{ut})^2}{S_e} \quad (5)$$

$$b = -\frac{1}{3} \log \left(\frac{fS_{ut}}{S_e} \right)$$

where f comes from Fig. 6-18. Then the safety factor in fatigue can be computed as

$$n_f = \frac{S_f}{\sigma_{rev}} \quad (6)$$

where

$$S_f = aN^b \quad (7)$$

Or, the number of cycles to failure can be computed as

$$N = \left(\frac{n_f \sigma_{rev}}{a} \right)^{1/b} \quad (8)$$

(g) Check for yielding. This is a straightforward check:

$$n_y = \frac{S_y}{\sigma_{rev}} \quad (9)$$

2. Fluctuating Simple Loading

- (a) Same as Case 1
- (b) Same as Case 1
- (c) Same as Case 1
- (d) Same as Item 1d, but instead of computing one reversal, both the midrange and alternating stresses need to be computed separately. Stress concentration factors should be applied to both.

$$\sigma_{m0} = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (10)$$

$$\sigma_{a0} = \frac{|\sigma_{max} - \sigma_{min}|}{2} \quad (11)$$

$$\sigma_{mmax} = K_f \sigma_{m0} \quad (12)$$

$$\sigma_{amax} = K_f \sigma_{a0} \quad (13)$$

- (e) Same as Item 1e, but instead apply one of the failure criteria for infinite life that accounts for midrange and alternate stress. We use Goodman's theory in this class. If $\sigma_m \geq 0$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f} \quad (14)$$

else if $\sigma_m < 0$

$$n_f = \frac{S_e}{\sigma_a} \quad (15)$$

If the stress is a torsional one, then you need to use a reduced ultimate strength in the Goodman calculations $S_{su} = 0.67S_{ut}$ (see Section 6-13 in Shigley).

- (f) If n_f is less than one, then there is a finite life and you need to determine the finite life number of cycles or factor of safety just as done in Item 1f. To use those equations you need to convert the midrange and alternating stresses into an equivalent fully reversed stress. As discussed in class, this is done for Goodman's theory using:

$$\sigma_{rev} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} \quad (16)$$

If you have pure torsion (shear stress), then you must use a reduced ultimate strength as described in Section 6-13 of Shigley. $S_{su} = 0.67S_{ut}$. Use this reduced S_{su} in replace of S_{ut} in Eq. (16) above.

- (g) Check for yielding. This is still straightforward.

$$n_y = \frac{S_y}{\sigma_m + \sigma_a} \quad (17)$$

If you have pure torsion (shear stress), you need to compare to the "shear yield strength" which is $S_{sy} = 0.577S_y$ (or equivalently convert the shear stress to a von Mises stress before comparing to the yield stress).

3. Combined Simple Loading

- (a) Same as Case 2
- (b) Same as Case 2, except do not apply k_c to the endurance limit because there are multiple loads. This correction will be used in step (d).
- (c) Same as Case 2
- (d) Same as Case 2, except that a von Mises calculation is needed for the midrange and alternating stress components separately. The equation is slightly complicated because the fatigue stress concentration factors differ for bending, axial loading, and torsion so be sure to apply each factor to the corresponding stress. The axial correction factor of 0.85, only applies to the alternating component. The bending correction factor is 1.0, and the Marin factor for torsional should not be used because it is already accounted for in the von Mises stress calculation. Split σ into σ_m and σ_a , similarly split τ into τ_m and τ_a . Then combine them into von Mises stresses for the mean and alternating stresses separately.

mean:

$$\begin{aligned} \sigma_m &= K_{fbending}\sigma_{m-bending} + K_{f_{axial}}\sigma_{m-axial} \\ \tau_m &= K_{ftorsion}\tau_{m-torsion} \\ \sigma'_m &= \sqrt{\sigma_m^2 + 3\tau_m^2} \end{aligned} \quad (18)$$

alternating:

$$\begin{aligned} \sigma_a &= K_{fbending}\sigma_{a-bending} + K_{f_{axial}}\frac{\sigma_{a-axial}}{0.85} \\ \tau_a &= K_{ftorsion}\tau_{a-torsion} \\ \sigma'_a &= \sqrt{\sigma_a^2 + 3\tau_a^2} \end{aligned} \quad (19)$$

- (e) Same as Case 2
- (f) Same as Case 2
- (g) Local yielding can be checked with the same formulas as in Item 2g (except using the von Mises stresses computed in step (d)).

$$n_y = \frac{S_y}{\sigma'_m + \sigma'_a} \quad (20)$$

This is conservative. More accurately yielding should be checked by first combining the mean and alternate stresses into σ_{max} and τ_{max} and then computing the corresponding maximum von Mises stress. However, because the simpler check is always conservative, and because we are computing those values anyway, it is easier just to use this check. If our safety factor is larger than 1, then we are safe. However, if it is less than 1, we may or may not be yielding and so need to check using the more accurate approach. There is no need to use the more accurate check if the simple solution already shows we are safe from yielding.