Linear Least Squares

ME EN 275
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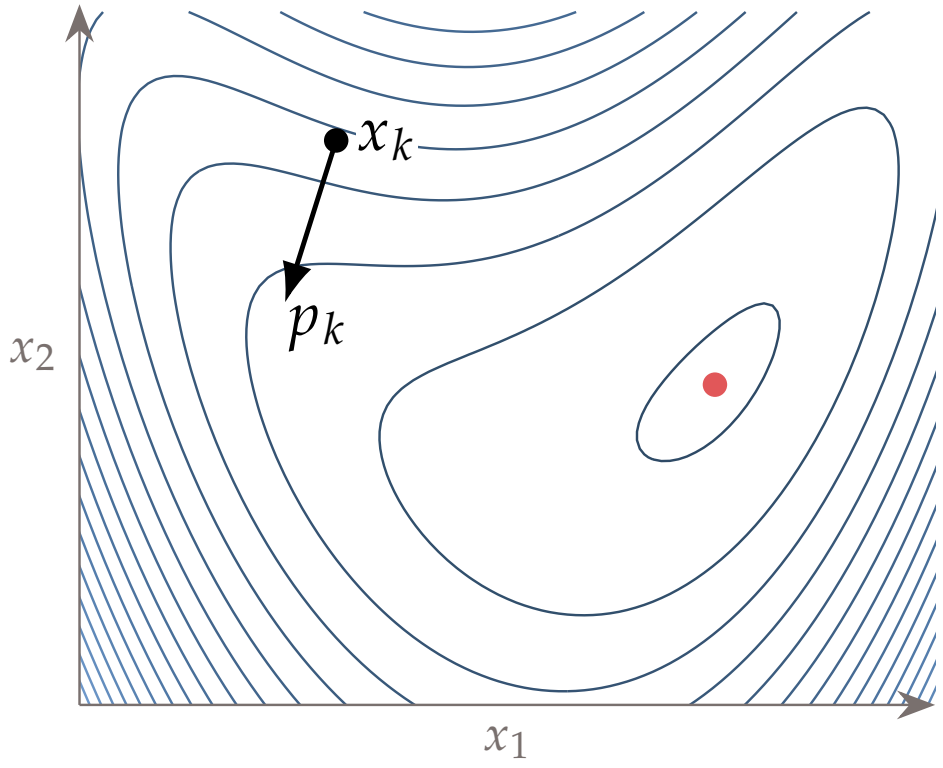


Alumni Achievement Award Lecture

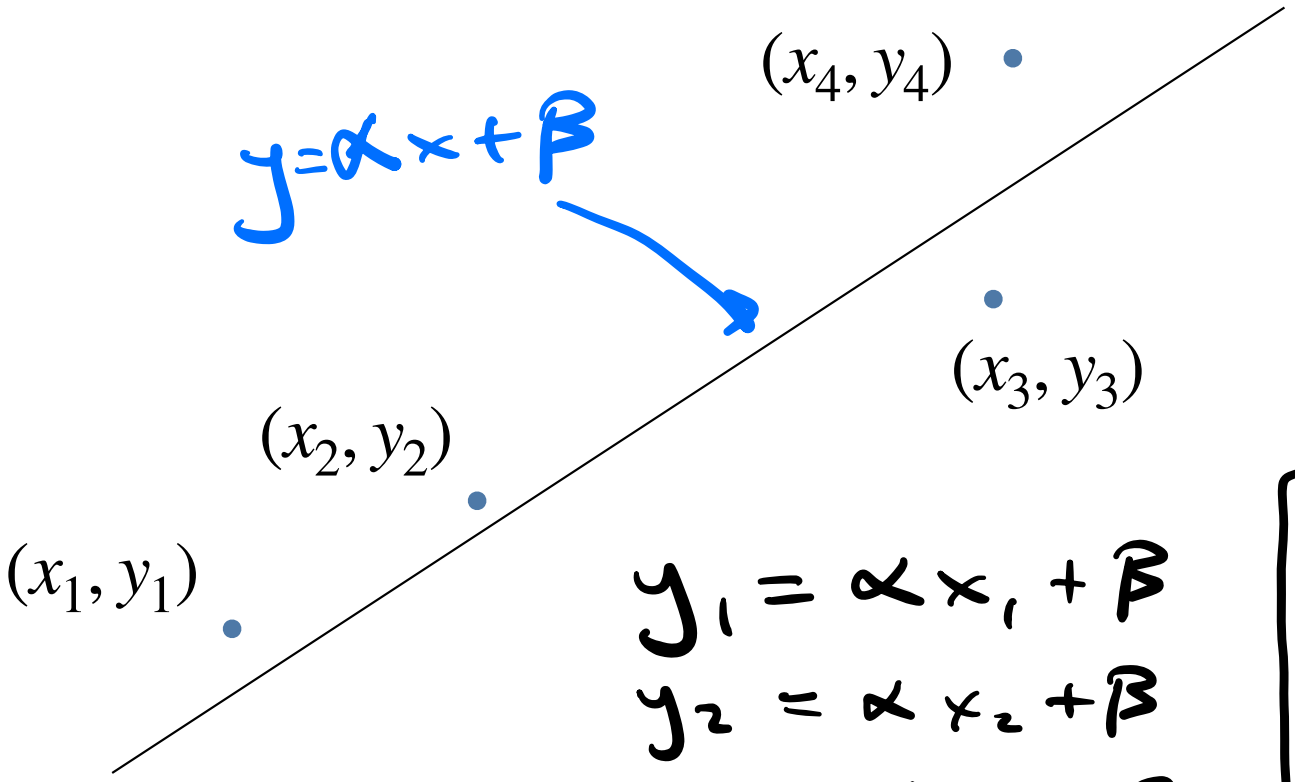
Thursday, October 10

11:00 AM - 12:00 PM
1170 TMCB

+Add to Calendar

BYU alumni Travis Oliphant

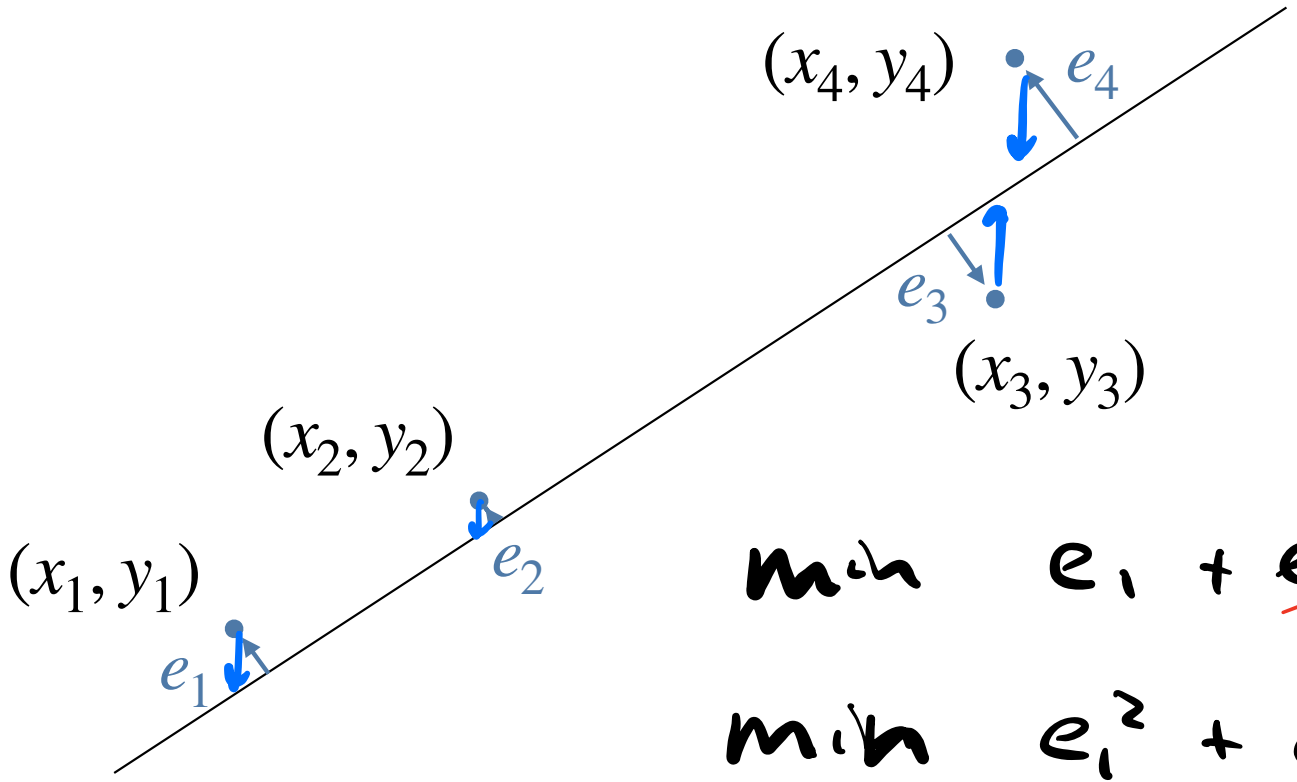
Motivation



$$
\begin{aligned}
y_1 &= \alpha x_1 + \beta \\
y_2 &= \alpha x_2 + \beta \\
y_3 &= \alpha x_3 + \beta \\
y_4 &= \alpha x_4 + \beta
\end{aligned}
$$

$$
\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \end{bmatrix}
\begin{bmatrix} \alpha \\ \beta \end{bmatrix}
=
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
$$

Least Squares

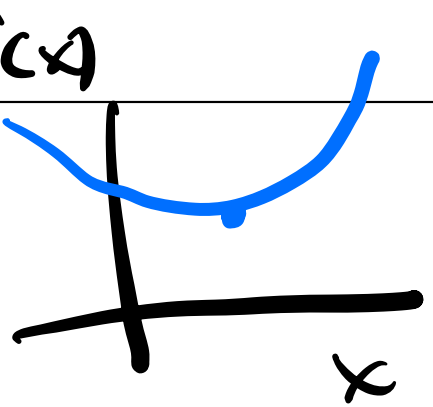


$$\min \quad e_1 + e_2 + e_3 + e_4$$

$$\min \quad e_1^2 + e_2^2 + e_3^2 + e_4^2$$

SSE sum of squared error.

Least Squares



$$\frac{df}{dx} = 0$$

$$y_1 = \alpha x_1 + \beta$$
$$y_2 = ax_2 + b$$
$$y_3 = ax_3 + b$$
$$y_4 = ax_4 + b$$

$$\min_E \rightarrow (\alpha x_1 + \beta - y_1)^2 + (\alpha x_2 + \beta - y_2)^2 + \cdots$$

$$\frac{dE}{d\alpha} = 0 \quad , \quad \frac{dE}{d\beta} = 0$$

Overdetermined

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

more eqs. than unknowns

Least Squares

$$e = Ax - b$$

$$\min \; e_1^2 + e_2^2 + e_3^2 + \dots$$

$$\min \; e^T e = (Ax-b)^T(Ax-b)$$

$$= x^T A^T A x - 2b^T A x + b^T b$$

derivative w.r.t.

$$2A^T A x - 2A^T b = 0$$

$$\Rightarrow \; x_{ls} = (A^T A)^{-1} A^T b$$

Least Squares

A 10×2

A^T 2×10

$A^T A$ 2×2

$$x_{ls} = \underbrace{(A^T A)^{-1} A^T}_{\text{psuedo inverse}} b$$

$$= A^+ b$$

```
np.linalg.lstsq(A, b)
```

Data Fitting Example

x	-1	0	1.5	3
y	3	1	3	8

fit: y = a x^2 + b x + c

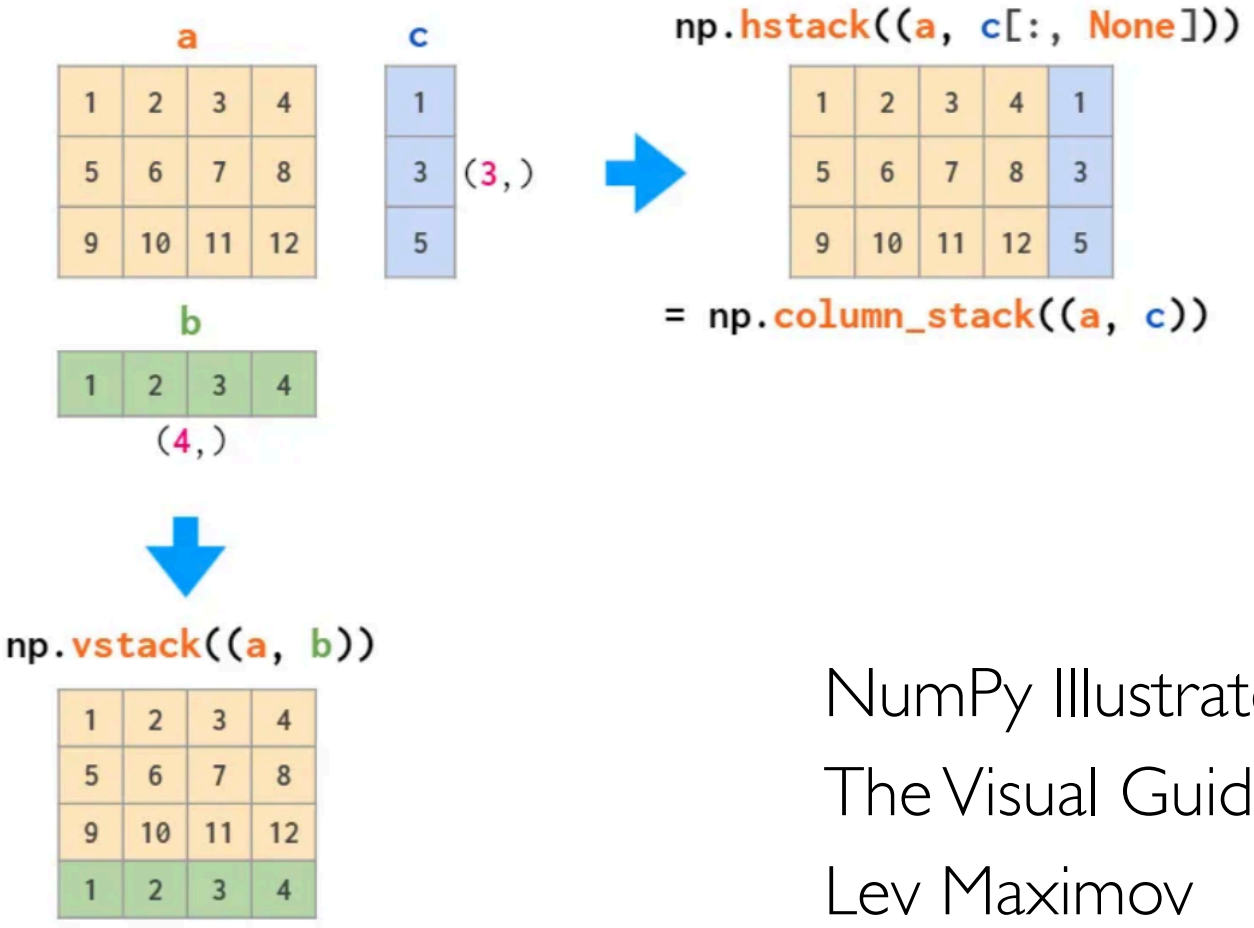
np.linalg.lstsq(A, b)

$$y_1 = a x_1^2 + b x_1 + c$$
$$y_2 = a x_2^2 + b x_2 + c$$
$$y_3 = a x_3^2 + b x_3 + c$$
$$y_4 = a x_4^2 + b x_4 + c$$

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix}$$

Data Fitting Example

Data Fitting Example



NumPy Illustrated:

The Visual Guide to NumPy

Lev Maximov