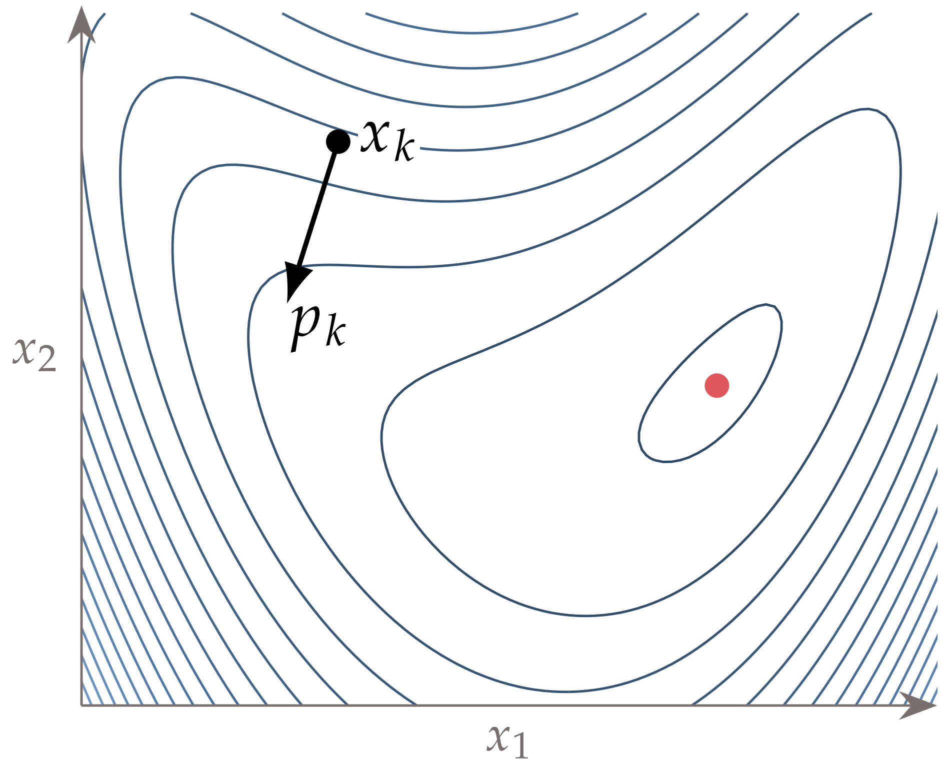
Linear Algebra



ME EN 275
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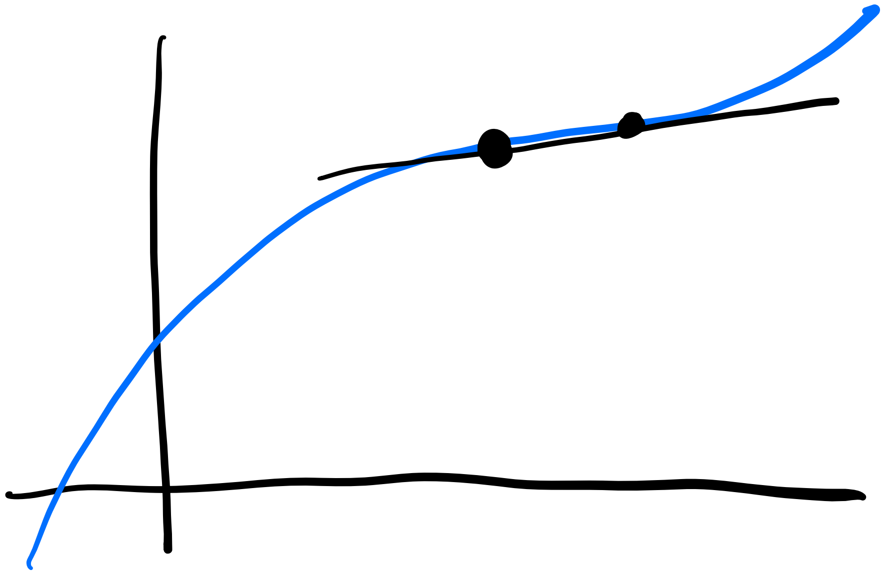
Differentiation Review



$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

Partial Derivatives

$f(x)$ x is a vector

$$\frac{\partial f}{\partial x_0} \qquad \frac{\partial f}{\partial x_1}$$

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example(x)
  a = x[0] + 3x[1]**2
  return a
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Motivation

Structures: finite element analysis

Dynamics: rotations, kinematics

Control: state space

Fluid Mechanics: low speed flows

EE: Circuit Analysis

Machine Learning

Linear Optimization

Vectors

$x = [1, 2, 3]$

row

$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

column

$x \leftarrow$ column $\qquad x^T \leftarrow$ row

x = np.array([1, 2, 3])

x.shape (3,)

y = x.reshape(3,1)

y.shape (3,1)

Dot Product

$x = [1, 2, 3]$

$y = [2, 1, 4]$

$\vec{x} \cdot \vec{y}$

$\sum_i x_i y_i$

$x^T y$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

np.dot(x, y)

Norm

$$\|\vec{x}\| \qquad \sqrt{\sum_i x_i^2} \qquad \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots}$$

np.linalg.norm(x)

Matrix-Vector Multiplication

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ \text{''} \\ \text{''} \end{bmatrix}$$

column vector

$$A \quad x \qquad\qquad y$$

row vector

$$[x_1, x_2\ x_3] \begin{bmatrix} & A & \end{bmatrix} = [\quad\quad]$$

np.dot(A, x)

np.dot(x, A)

Matrix-Matrix Multiplication

$$A \cdot B$$

np.dot(A, B)

np.matmul(A, B)

A @ B

Linear System of Equations

$$4x + 3y + 5z = 2$$
$$7x + y + 8z = 1$$
$$8x + 5y + 6z = 0$$

$$\begin{bmatrix} 4 & 3 & 5 \\ 7 & 1 & 8 \\ 8 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{A x = b}$$

Linear System of Equations

$$Ax = b \Rightarrow x = A^{-1} b$$

~~x = np.linalg.inv(A) @ b~~

x = np.linalg.solve(A, b)

Triangular System

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$a_{22} x_2 + a_{23} x_3 = b_2$$

$$a_{33} x_3 = b_3$$

$$A \quad x \quad = b$$

LU Decomposition

$$A = LU$$

$$\underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}$$

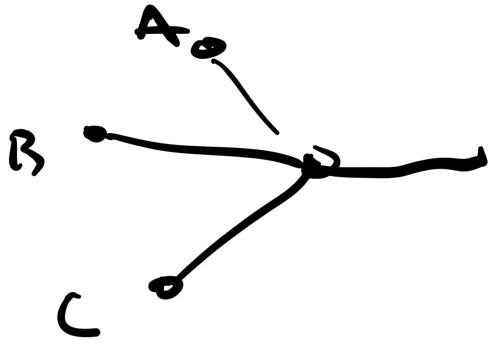
$$Ax = b$$

$$L\underbrace{Ux}_{y} = b \Rightarrow$$

1) $\check{L}y = \check{b} \Rightarrow y$

2) $\check{U}x = \check{y} \Rightarrow x$

Statics: Try it



$$-0.27F_A + 0.32F_B - 0.16F_C = 0$$

$$-0.8F_A - 0.95F_B - 0.97F_C + 5 = 0$$

$$-0.53F_A + 0.16F_C = 0$$

$Ax = b$

np.linalg.solve(A, b)