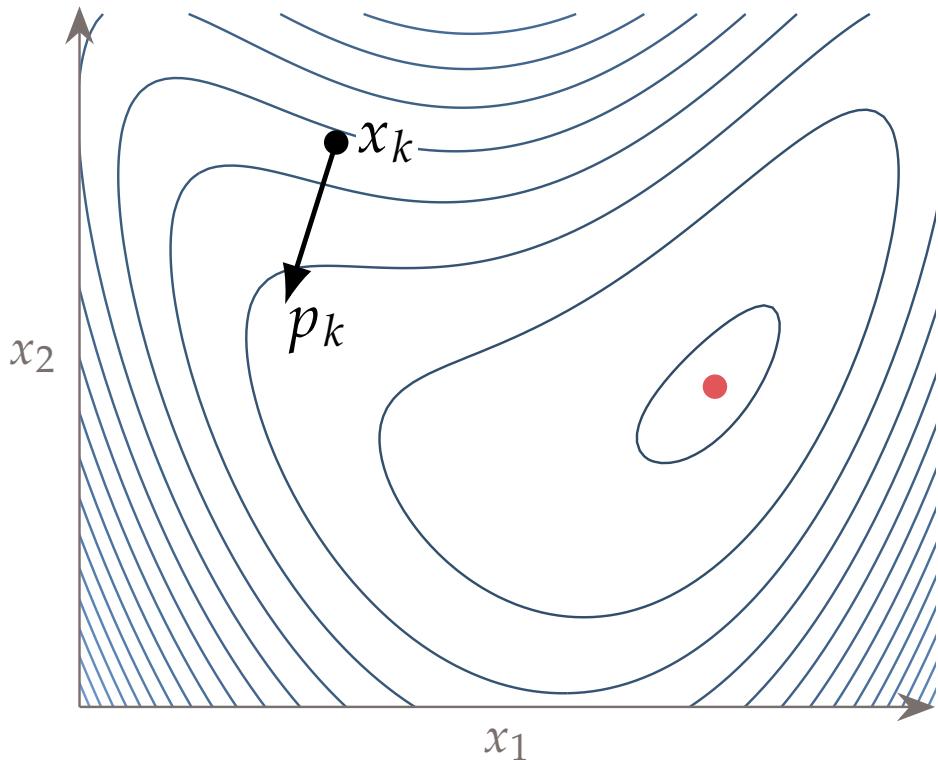


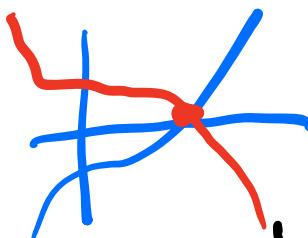
# Numerical Integrals



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# Root Finding Review

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$$y = x^2 \sin x \quad (\text{explicit})$$

$$y = x^2 \sin(x+y) \quad (\text{implicit})$$

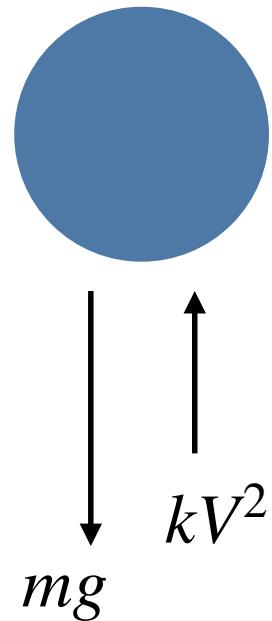
$$f(y, x) = g(y, x) \rightarrow y - x^2 \sin(x+y) = 0$$

$$\underbrace{f(y, x) - g(y, x)}_{r(y)} = 0$$

$$r(y) = 0$$

# Scipy Demo

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$$V = \sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{kg}{m}} t\right)$$

$$V - \sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{kg}{m}} t\right) = 0$$

A graph showing a blue curve representing the potential energy  $V$  over time  $t$ . The curve starts at a positive value, decreases towards zero, and then oscillates around the zero line. The label  $r(m)$  is written below the curve.

# Where We Are

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Algebra

Calculus

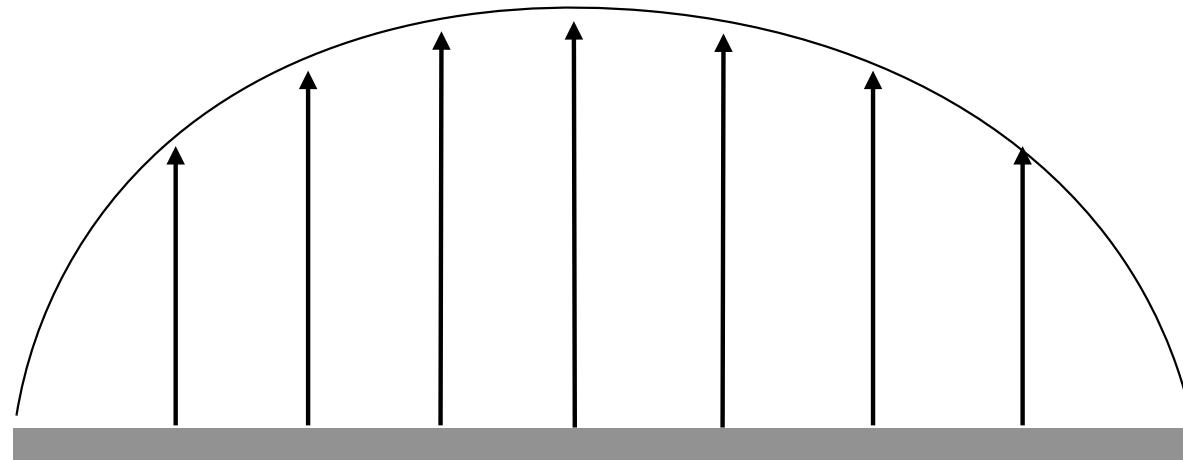
root finding

integration

differentiation

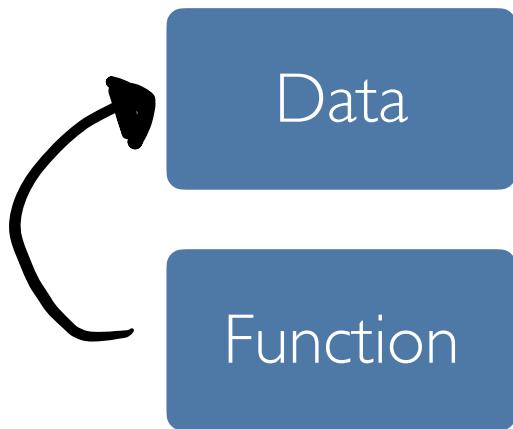
# Motivation

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## Two Types

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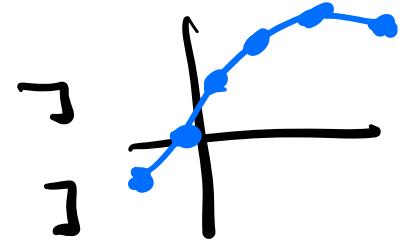
*x is given*

$x = [x_1, x_2, \dots]$

$f = [f_1, f_2, \dots]$

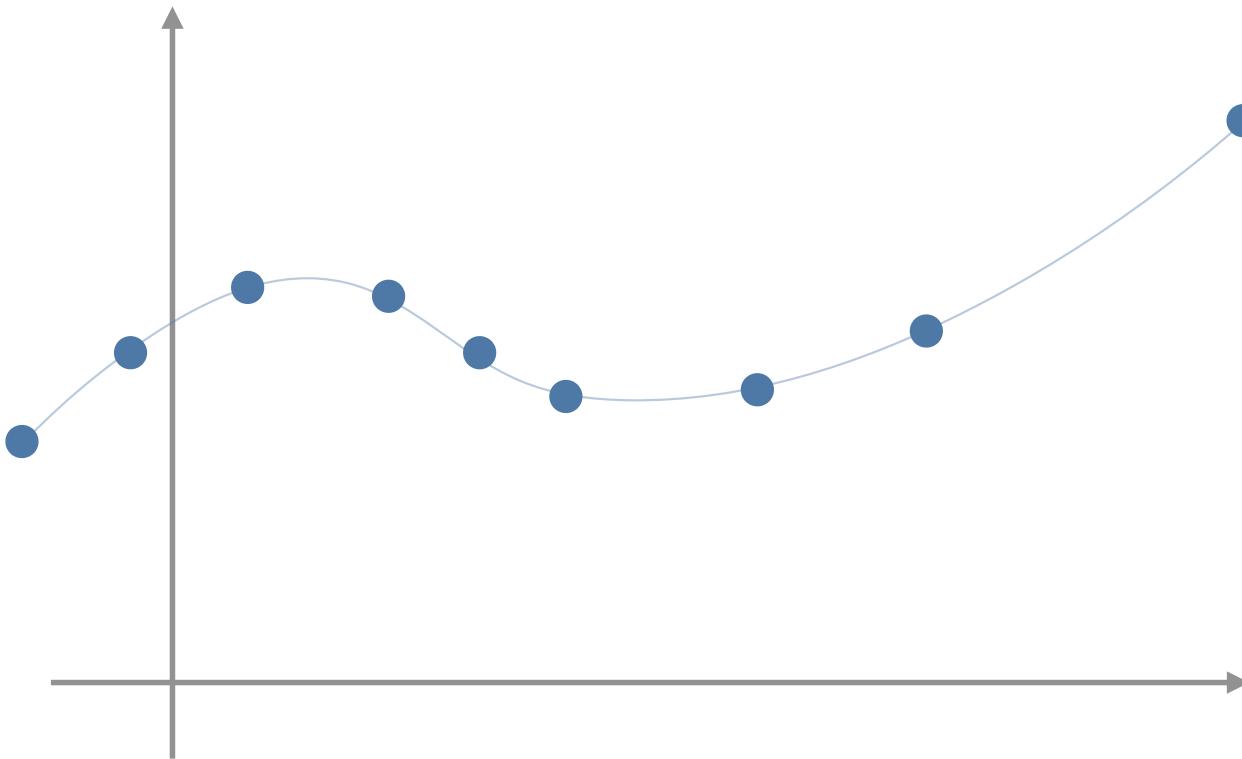
*f(x)*

*x I can choose*



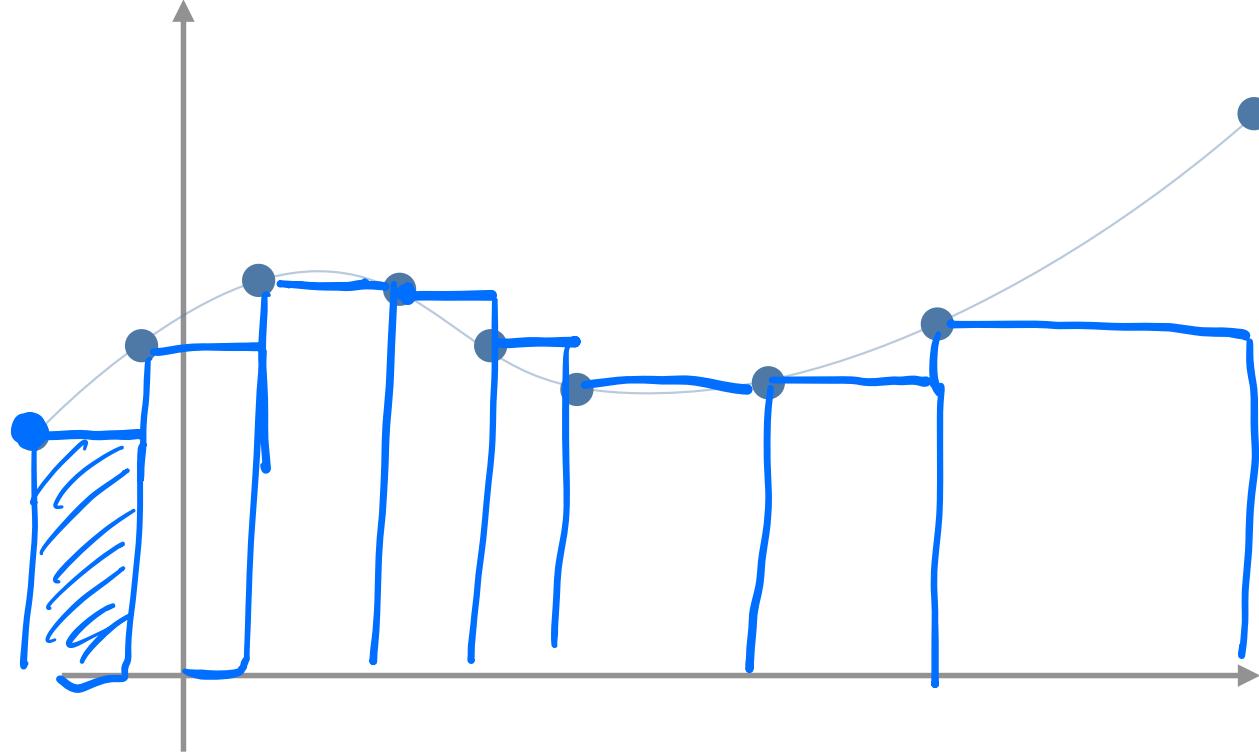
How would you do it?

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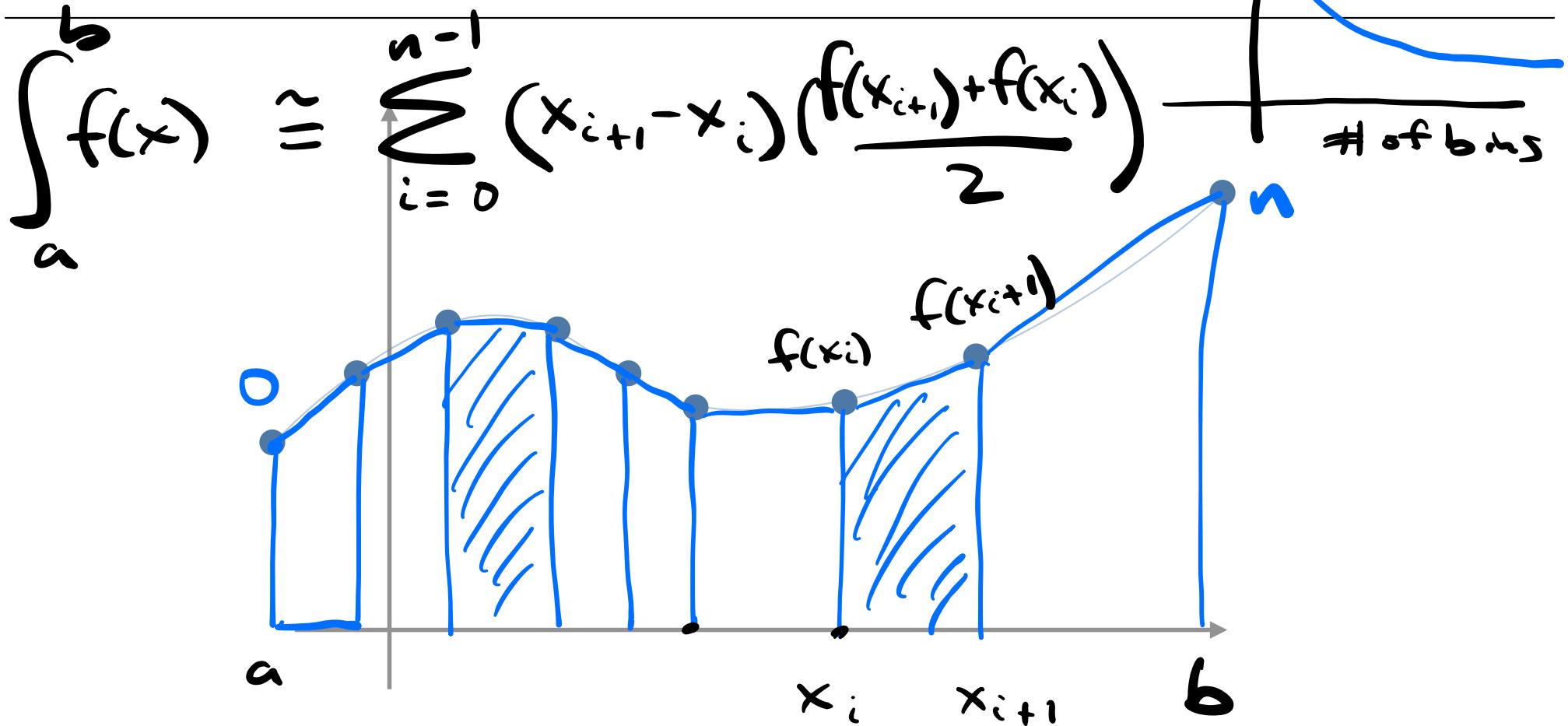


# Rectangle rule

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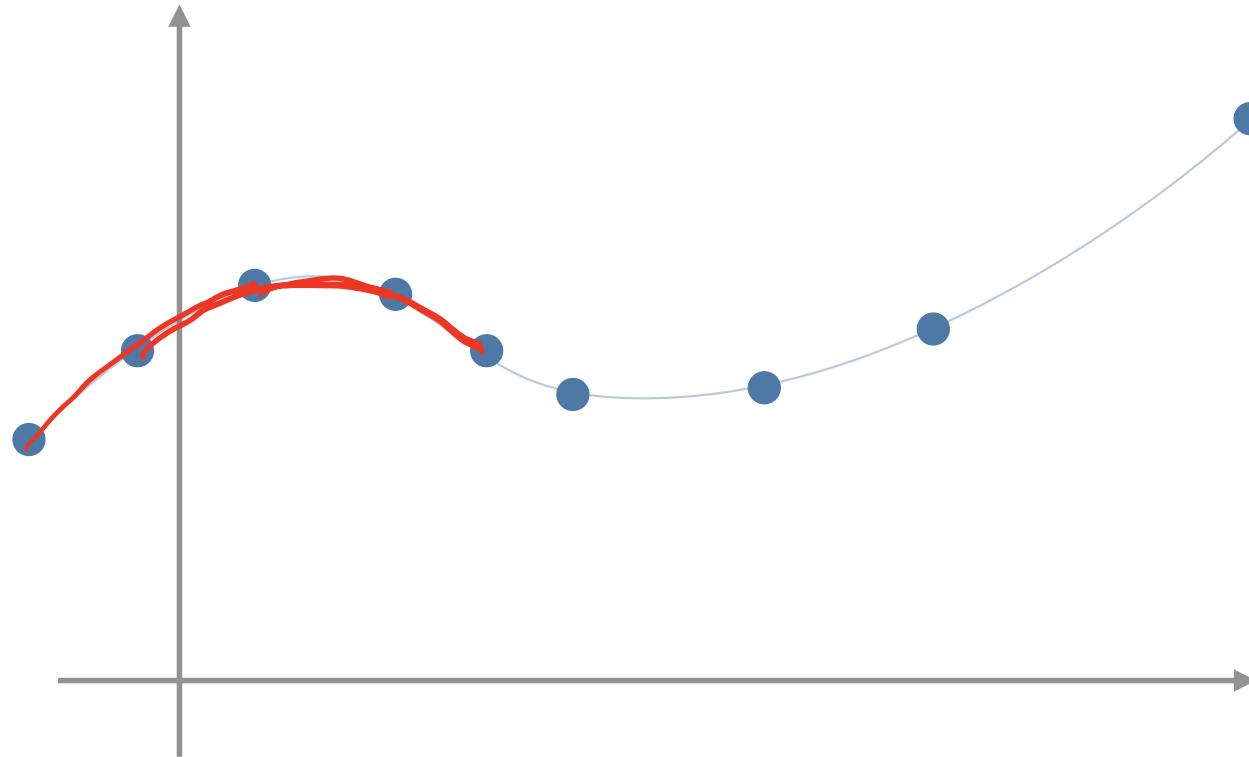


## Trapezoidal rule



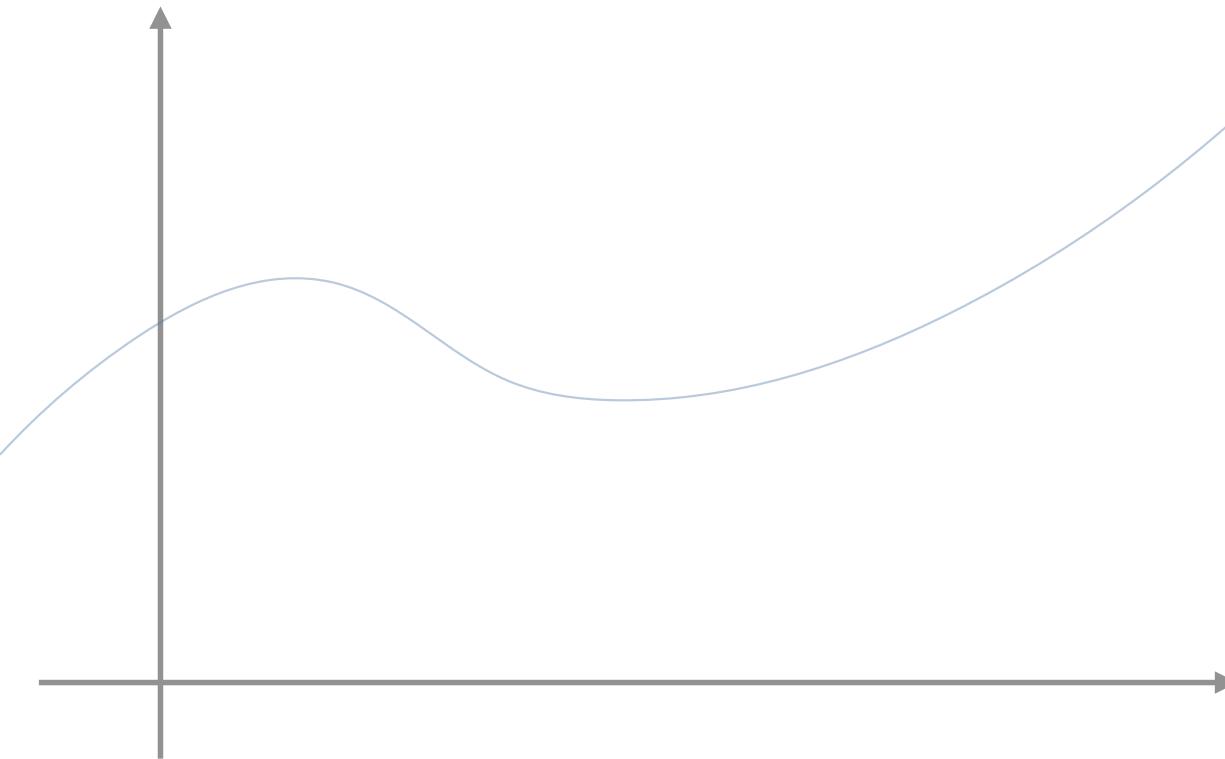
# Simpson's rule

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# Gaussian quadrature

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Let's try it

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$$\int_0^{\pi} \sin(x^2)$$

np.trapz

← trapezoidal

or np.trapezoid

~~scipy.optimize.quad~~

← Gaussian  
quad rule.

integrate