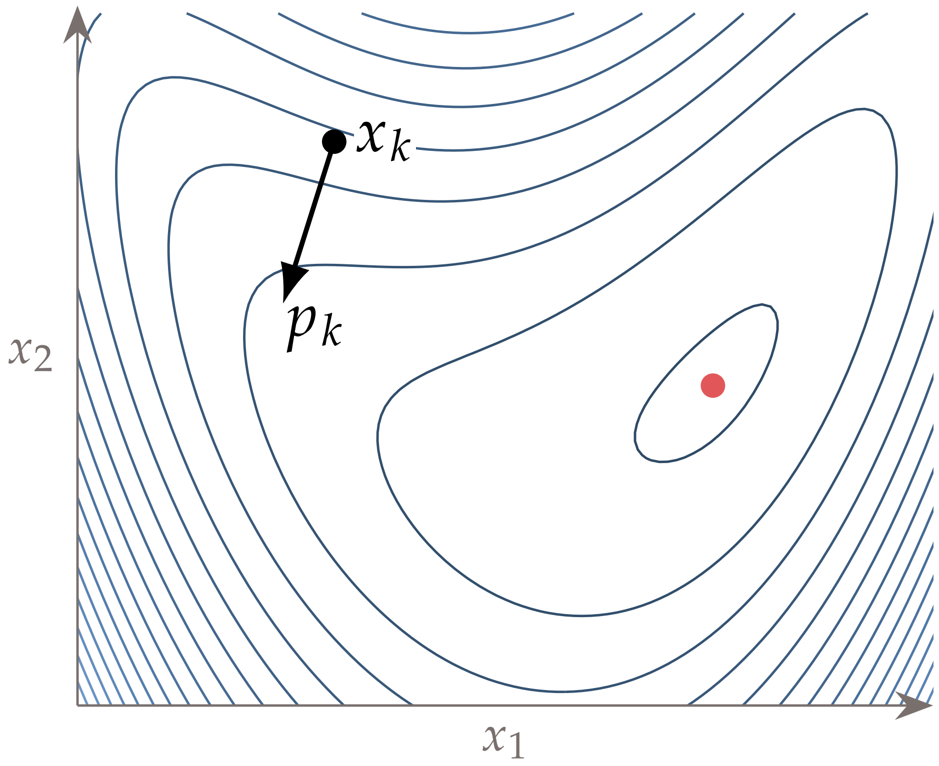
Confidence Intervals



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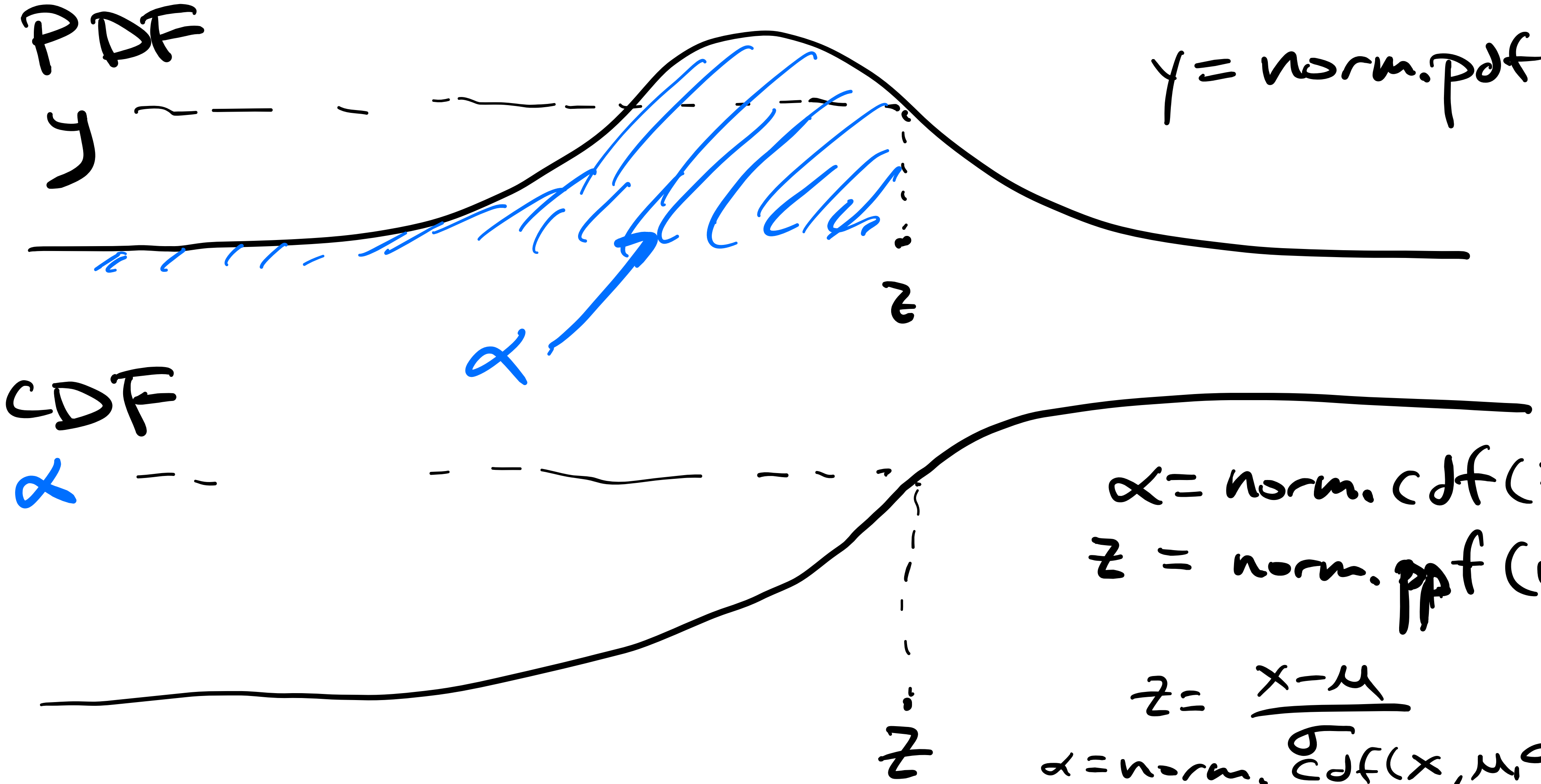
CDF and PPF (inverse CDF)

from scipy.stats import norm



$y = \text{norm.pdf}(z)$

$\alpha = \text{norm.cdf}(z)$

$z = \text{norm.ppf}(\alpha)$

$z = \frac{x-\mu}{\sigma}$

$\alpha = \text{norm.cdf}(x, \mu, \sigma)$

Example

scipy.stats.norm. cdf
ppf

Battery lifetimes for a given application are normally distributed with mean 50 hrs and stdev = 5 hrs.

What is probability that a random battery lasts longer than 52 hours?

What is probability that a random battery lasts between 42 and 52 hours?

What is the 40th percentile of battery lifetimes?

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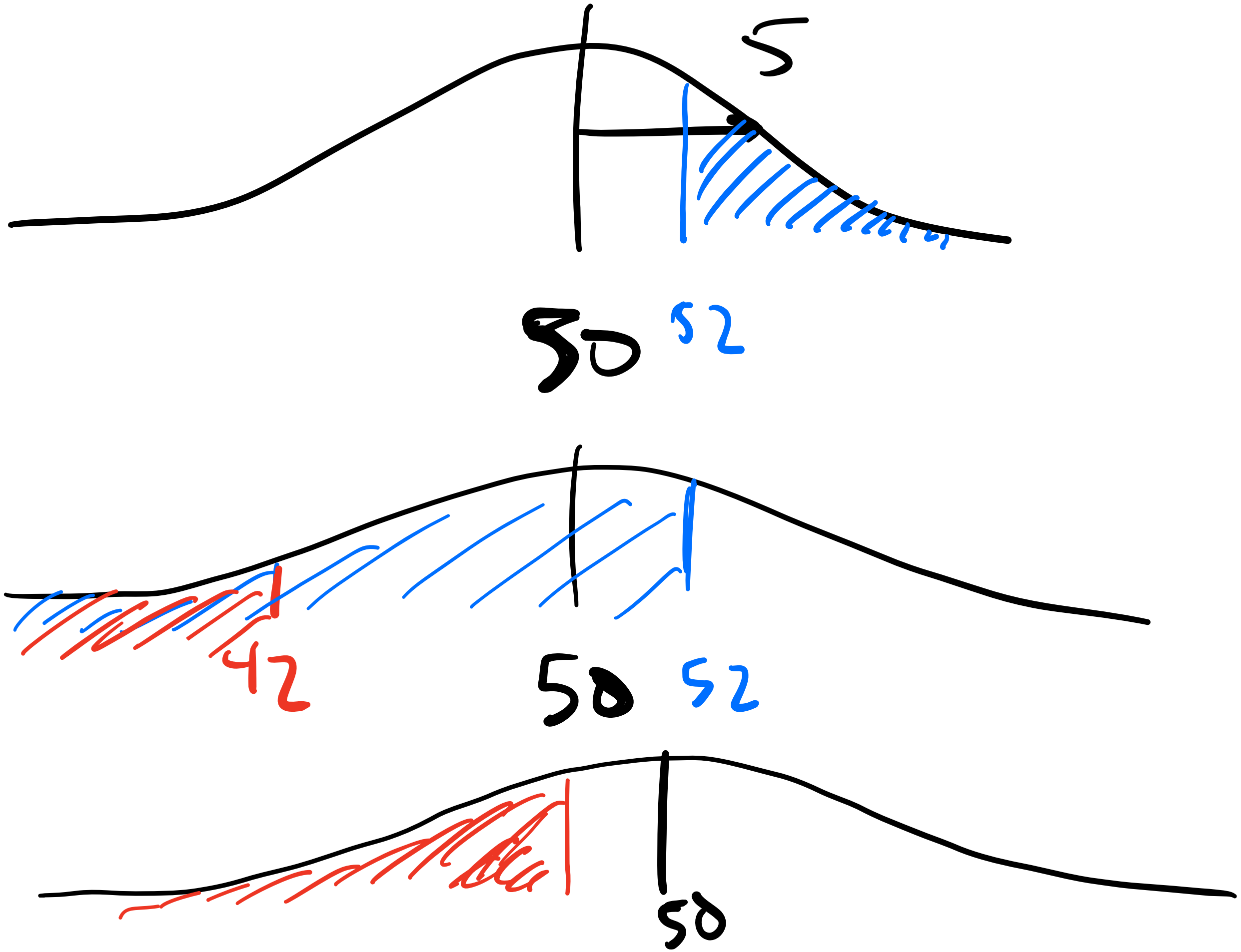
Example

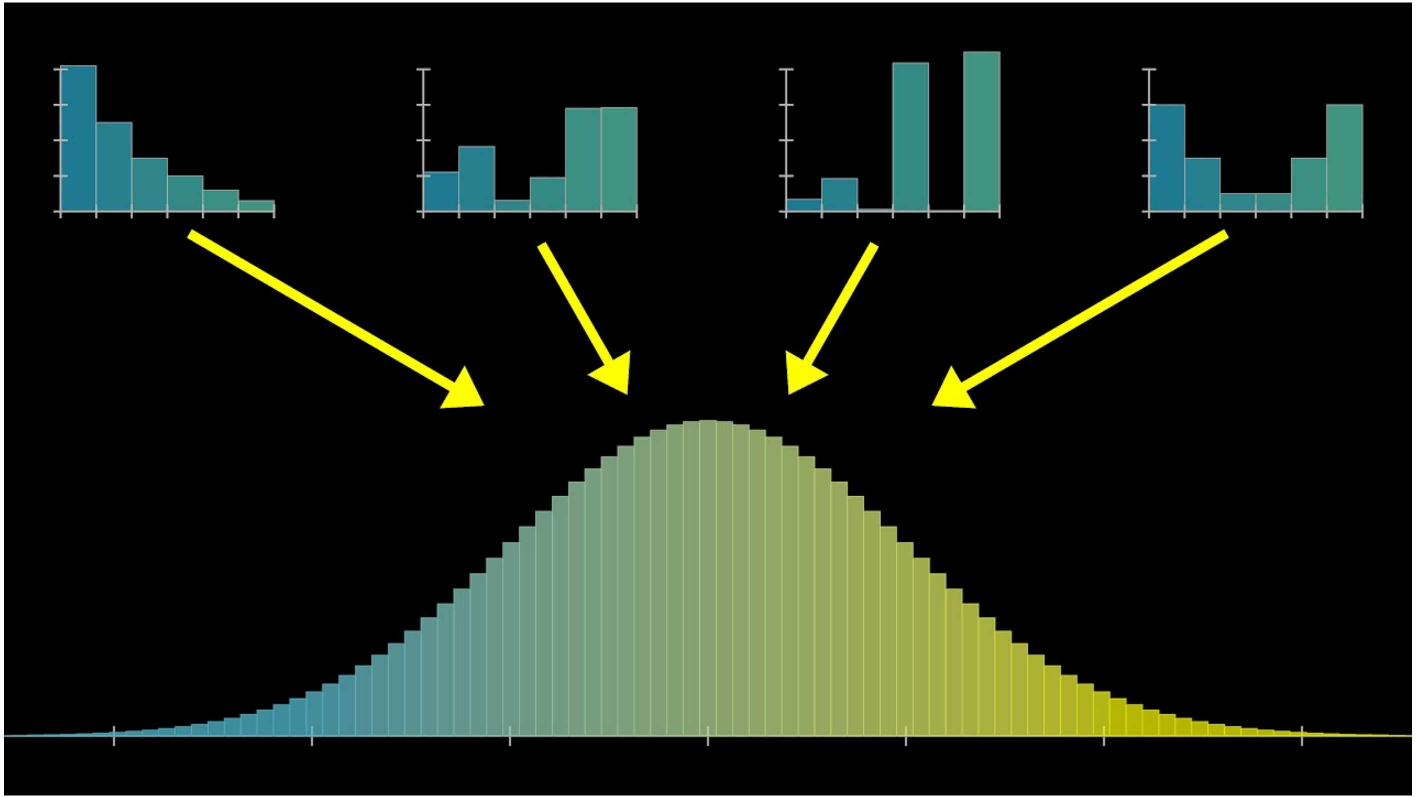


Other Common Distributions

https://en.wikipedia.org/wiki/List_of_probability_distributions

Motivation

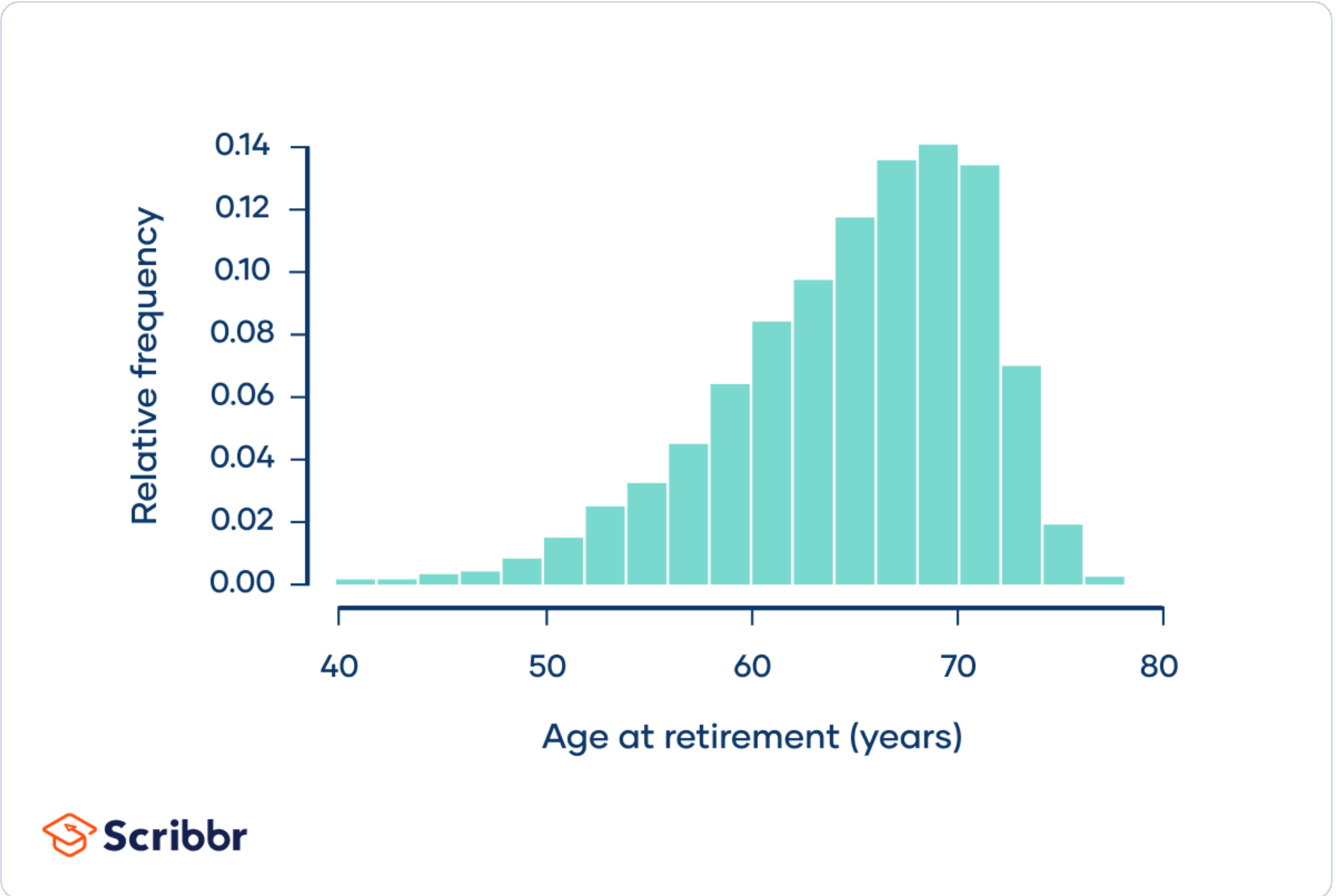
Suppose I sampled the height of 100 people in Provo, UT computed their mean height. I would expect the mean I measured to not be quite the true mean of all people in Provo - perhaps higher or lower. But how good is my estimate? Can I provide a reasonable range for what the mean height is?



Central Limit Theorem

sampling distribution of the mean will always be normally distributed (assuming sample is large enough, say > 30)

Example (from scribbr.com)



68	73	70	62	63

mean = (68 + 73 + 70 + 62 + 63) / 5

mean = 67.2 years

sampling distribution of the mean

60.8	57.8	62.2	68.6	67.4	67.8	68.3	65.6	66.5	62.1

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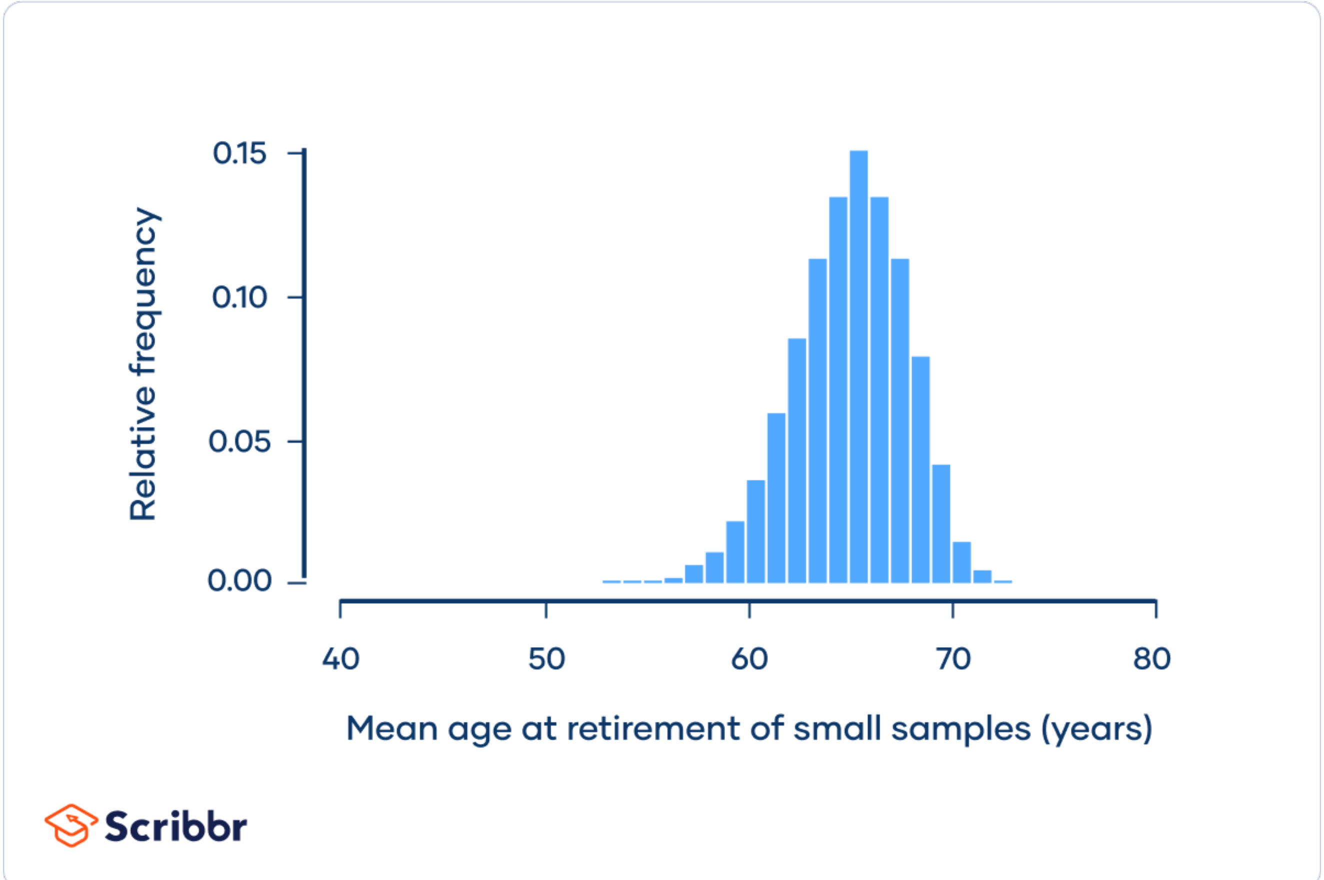
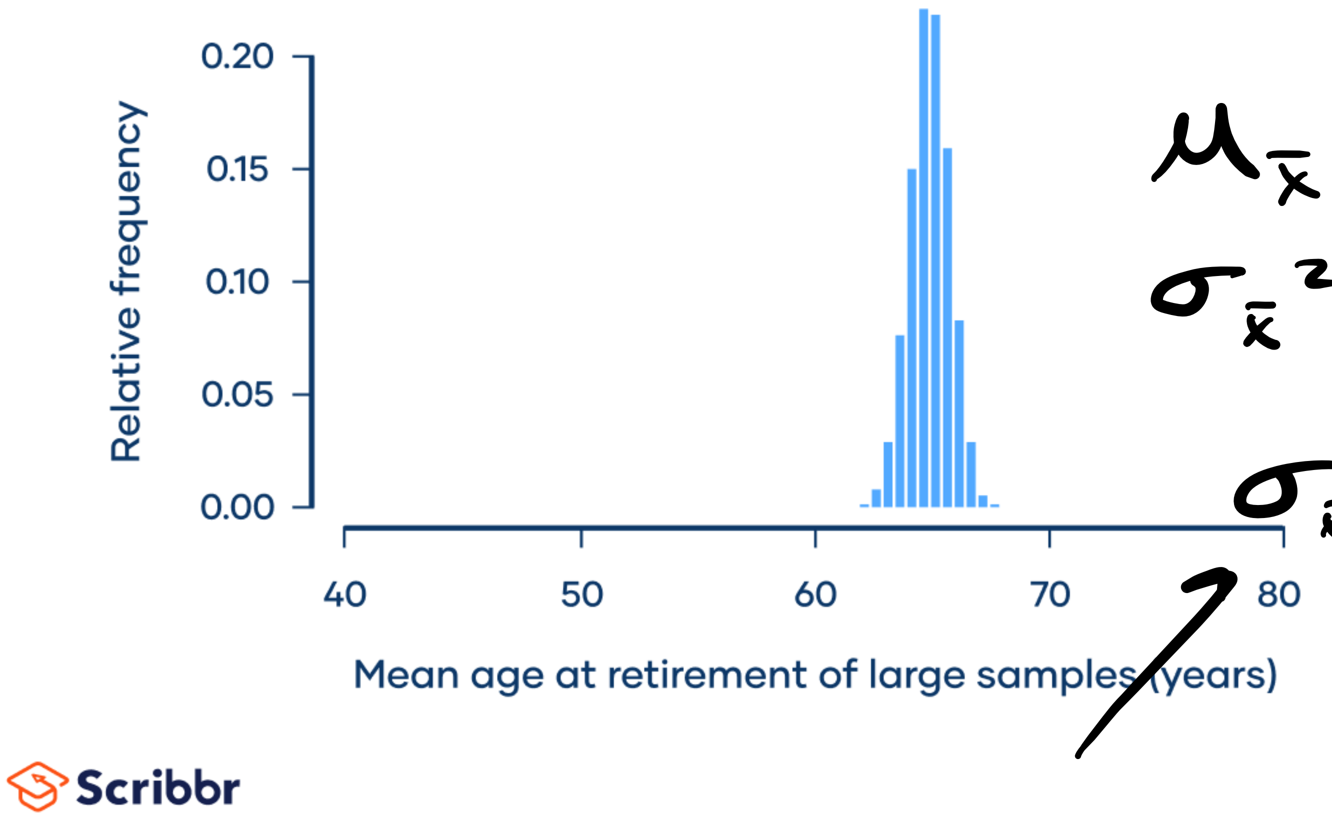


73	49	62	68	72	71	65	60	69	61
62	75	66	63	66	68	76	68	54	74
68	60	72	63	57	64	65	59	72	52
52	72	69	62	68	64	60	65	53	69
59	68	67	71	69	70	52	62	64	68

mean = 64.8 years

$$\bar{x} = \frac{x_1 + x_2 + \dots x_n}{n}$$



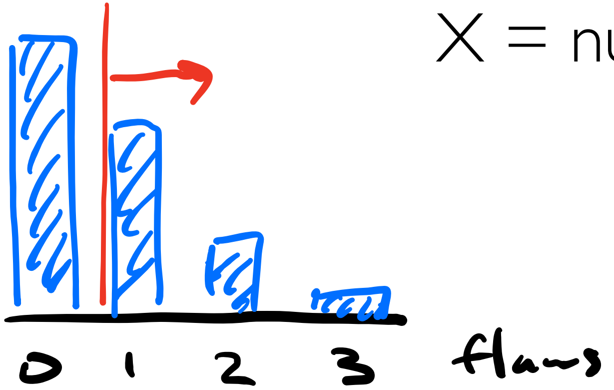
$$\mu_{\bar{x}} = \mu_x$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

standard error = stdev of sample mean

Example



X = number of flaws in a copper wire

$\mu_{\bar{x}} = \mu_x$

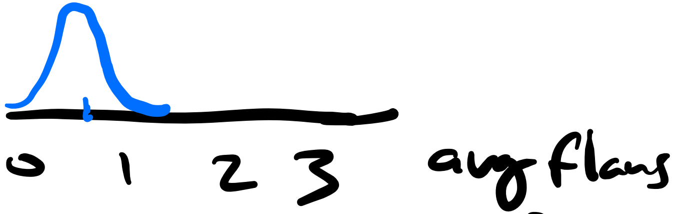
$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$

x	$P(X = x)$
0	0.48
1	0.39
2	0.12
3	0.01

$\mu_x = 0(0.48) + (1)(0.39) + \cdots$
$= 0.66$

$\mu_{\bar{x}} = 0.66$

$\sigma_{\bar{x}} = \sqrt{.5244}/\sqrt{100}$



from samples of n = 100

If I sample 100 of them, what is probability that they will have an average number of flaws less than 0.5?

$\sigma_x^2 = (0-\mu_x)^2(0.48) + (1-\mu_x)^2(0.39) + \cdots$
$= 0.5244$

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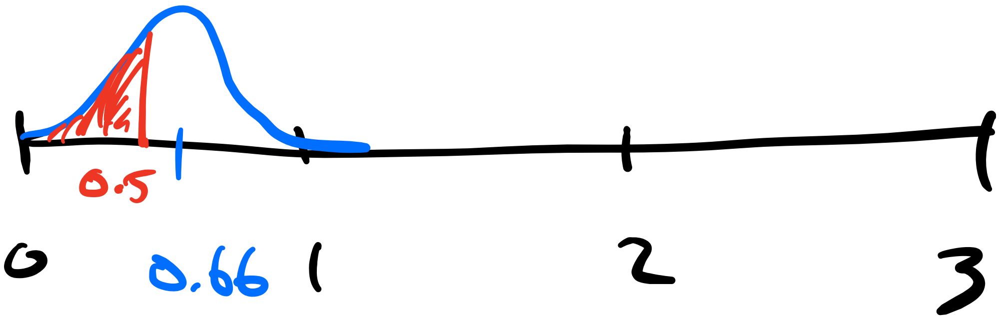
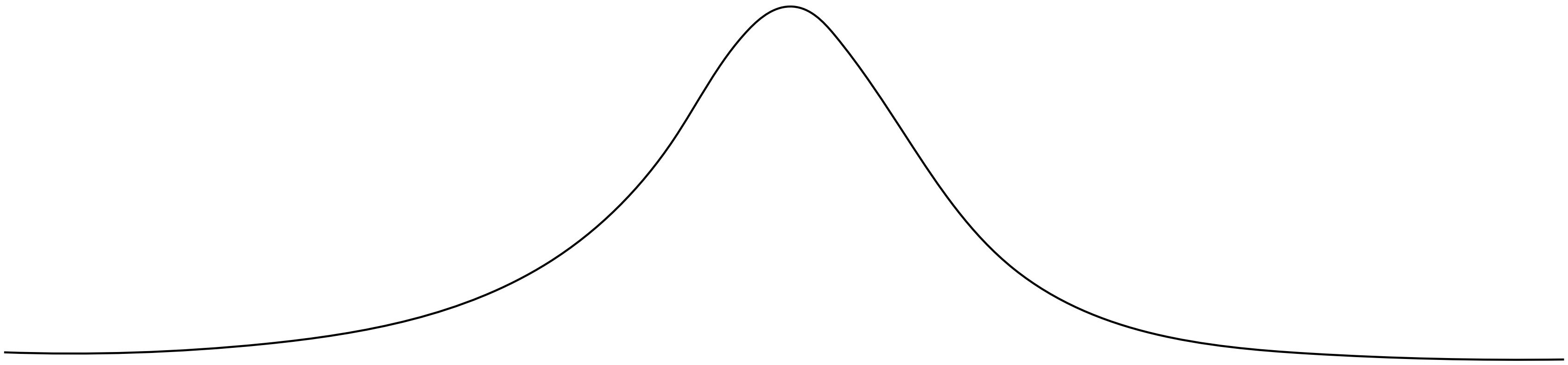
Example

$$\text{norm.cdf}\left(0.5, 0.66, \sqrt{\frac{.5244}{100}}\right)$$

$\sim 1.35\%$



Example (heights)



Other confidence intervals

Example

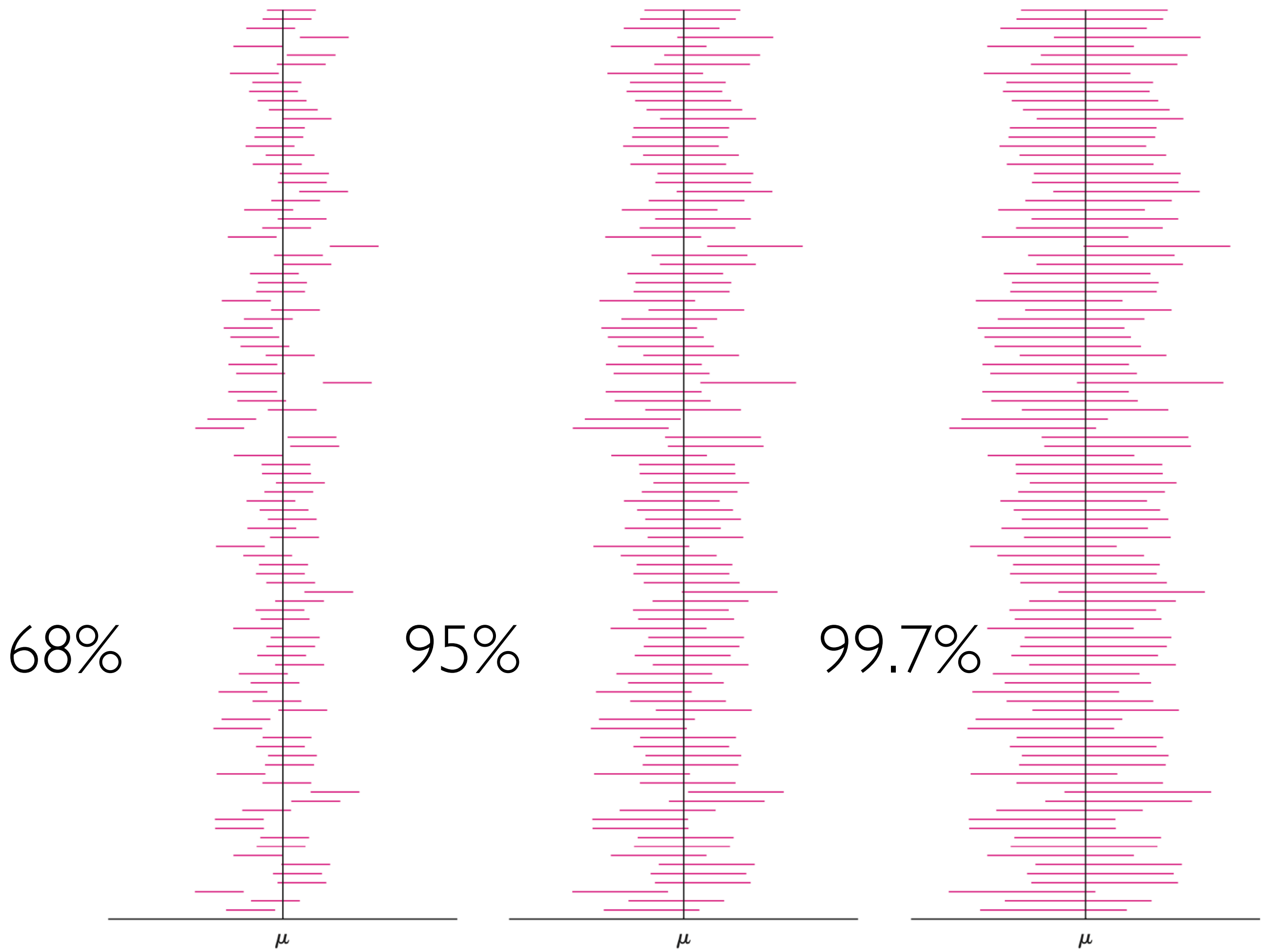
sample size = 50

$\bar{x} = 12$

$s_x = 1$

find 80% confidence interval

What confidence level should I use?



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