Probability

ME EN 275
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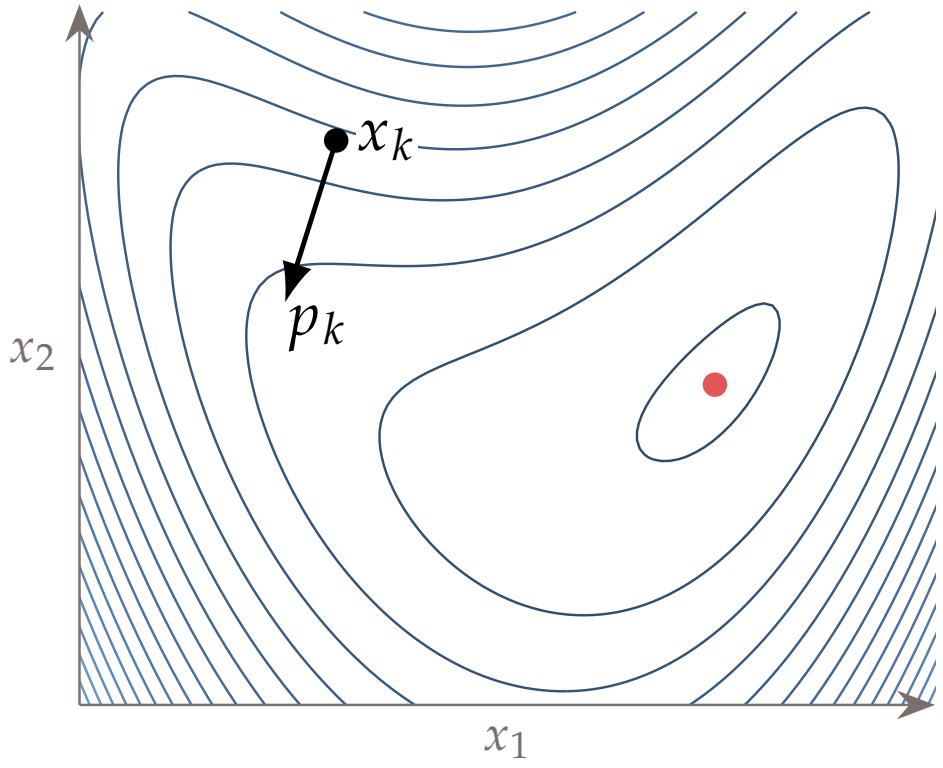


Birthday Problem

60 people in this room, what is the probability
that at least two of us have the same birthday?

Probability

$$\text{probability} = \frac{\#\text{ of positive outcomes}}{\#\text{ of possible outcomes}}$$

equally likely.

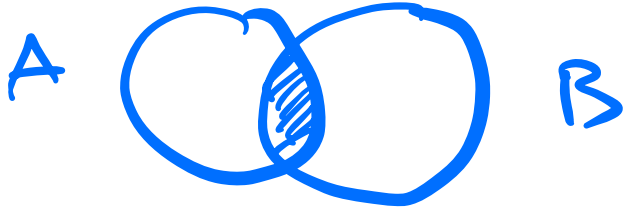
Combining Events

Union: $A \cup B$

A or B

Intersection: $A \cap B$

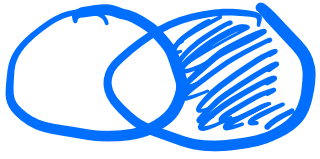
A and B



Complementary A^c

not A

(not A) and B



Probability Rules

$$0 \leq P(A) \leq 1$$

$$P(\text{ not } A) = 1 - P(A)$$

Independence

if independent $P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

Examples

If you flip a coin twice, what is the probability that it will come up heads both times?

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

If you flip a coin and roll a six-sided die, what is the probability that the coin comes up heads and the die comes up 1?

$$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

You draw a card from a deck of cards, put it back, and then draw another card. What is the probability that the first card is a heart and the second card is black?

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

Mutually Exclusive

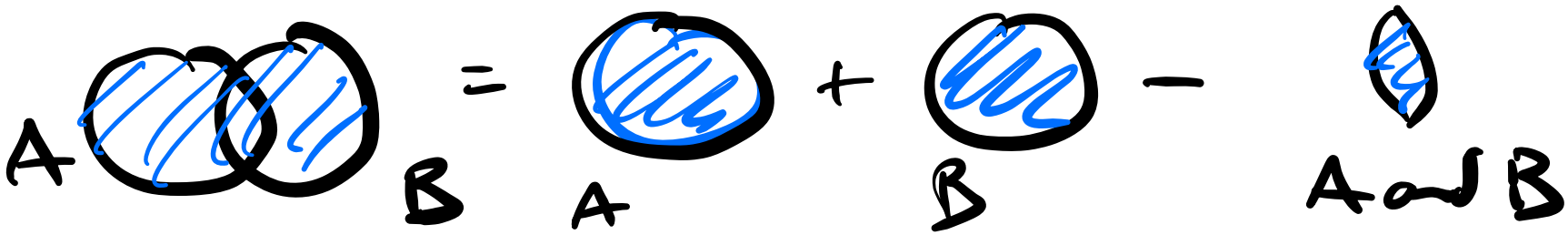
independent: events do not affect each other's outcomes.

mutually exclusive: events that cannot occur at the same time.

if mutually exclusive $P(A \text{ or } B) = P(A) + P(B)$

Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Examples

Flip a coin twice. Probability you get at least one head

$$P(\underset{A}{H1} \text{ or } \underset{B}{H2}) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

Flip a coin and roll a die. Probability that you get a 6 or a head.

$$P(6 \text{ or } H) = \frac{1}{6} + \frac{1}{2} - \frac{1}{6}\cdot\frac{1}{2} = \frac{7}{12}$$

Examples

If you throw a die three times, what is the probability that you roll a 2 at least once.

$$P(2R1 \text{ or } 2R2 \text{ or } 2R3)$$

$$1 - P(\text{no } 2\text{s})$$

$$1 - \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 0.42$$

Conditional Probability

$P(\text{diameter is ok}) = \frac{928}{1000} = .928$

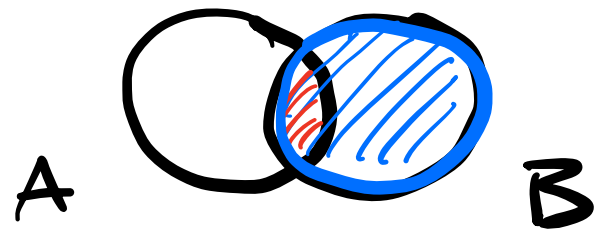
$P(\text{diameter is ok} \mid \text{length too long}) = \frac{25}{40} = .625$

	Diameter		
Length	**Too Thin**	**OK**	**Too Thick**
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

$P(A \mid B)$ probability of A given B / conditioned on B

Conditional Probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{25/100}{44/100}$$



Conditional Probability

what is the probability that two cards drawn at random from a deck of playing cards will both be aces?

$$P(A1 \text{ and } A2) = \frac{4}{52} \cdot \frac{3}{51}$$

If you draw two cards from a deck, what is the probability that you will get the Ace of Diamonds and a black card?

$$\frac{1}{52} \cdot \frac{26}{51} + \frac{1}{2} \cdot \frac{1}{51} = \frac{1}{51}$$

Multiplication Rule

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$\hookrightarrow P(A \text{ and } B) = P(A|B) \cdot P(B)$$

$$= P(B|A) \cdot P(A)$$

if independent $P(A|B) = P(A)$

$$\hookrightarrow P(A \text{ and } B) = P(A) \cdot P(B)$$

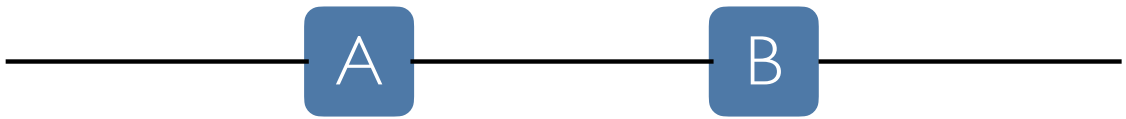
Birthday Problem

60 people in this room, what is the probability that at least two of us have the same birthday?

$1 - P(\text{no 2 people w/ same b-thdy})$.

$$1 - \left[1 \cdot \frac{364}{365} \cdot \frac{365}{365} \cdot \dots\right]$$

Reliability Analysis



Reliability Analysis