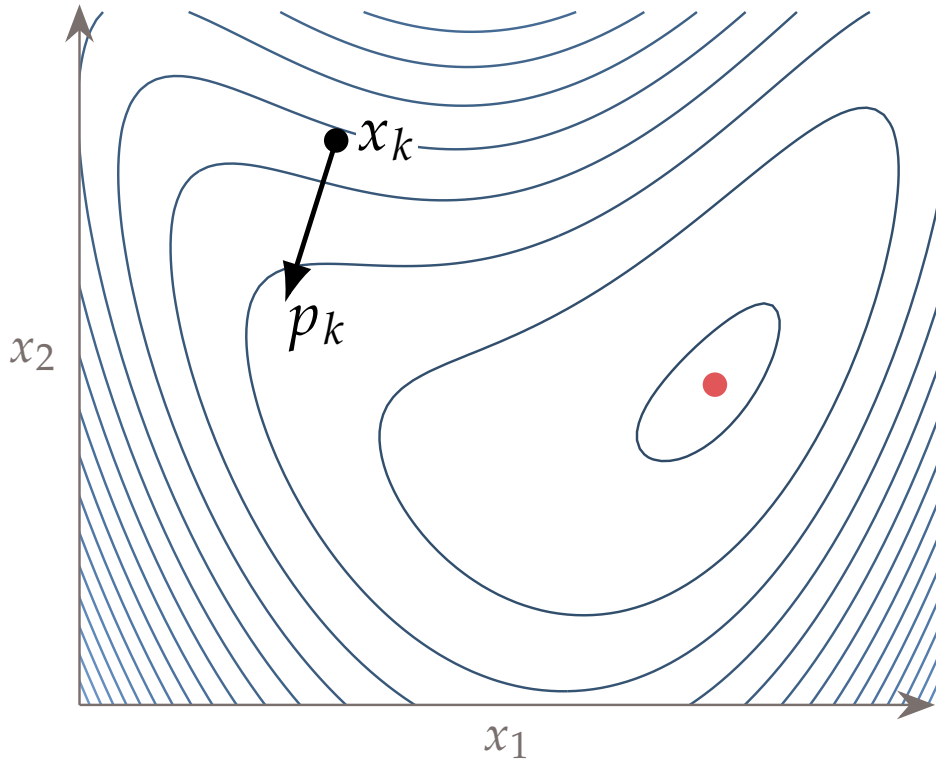
ODE Examples

ME EN 275
Andrew Ning
aning@byu.edu



First Order ODE

$$\frac{dy}{dx} - 2xy = 0$$

$$y(0) = 1$$

$$x = -2 \ldots 2$$

$$\frac{dy}{dx} = 2xy$$

$$y(0) = 1$$

scipy.integrate.solve_ivp

Predator Prey (Coupled First-Order)

$$\frac{dy}{dt} = f(t, y)$$

$$y = [u, v]$$

$$\frac{du}{dt} = \alpha u - \beta u v \quad \text{(prey)}$$

$$\frac{dv}{dt} = -\gamma v + \delta u v \quad \text{(predator)}$$

$$\alpha = 1, \gamma = 1, \beta = 0.01, \delta = 0.02$$

$$t = 0 \ldots 15$$

$$u(0) = 400$$

$$v(0) = 100$$

Higher Order ODEs

$$\frac{d^{(n)}y}{dt^{(n)}} = f\left(t, y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \dots\right)$$

$$y^{(n)} = f(t, y, y', y'', y''', \dots)$$

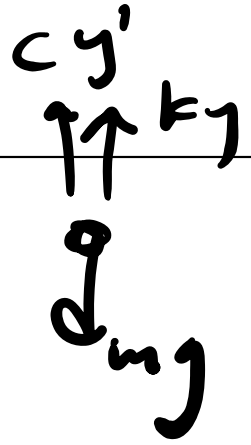
$$z_1 = y \qquad z_1' = z_2$$

$$z_2 = y' \quad \Rightarrow \quad z_2' = z_3$$

$$z_3 = y'' \qquad z_3' = f(t, z_1, z_2, z_3)$$

$$\vdots$$

Bungee jump



$$my'' + cy' + ky = mg$$

$$y'' = g - \frac{c}{m}y' - \frac{k}{m}y$$

$z_1 = y$

$z_2 = y'$

$\Rightarrow$

$$z_1' = z_2$$

$$z_2' = g - \frac{c}{m}z_2 - \frac{k}{m}z_1$$

$g = 9.81$

$m = 80.0$

$k = 10.0$

$c = 20.0$

$t = 0 \ldots 100$

start at 0 from rest