Ordinary Differential Equations (part 2)

ME EN 275
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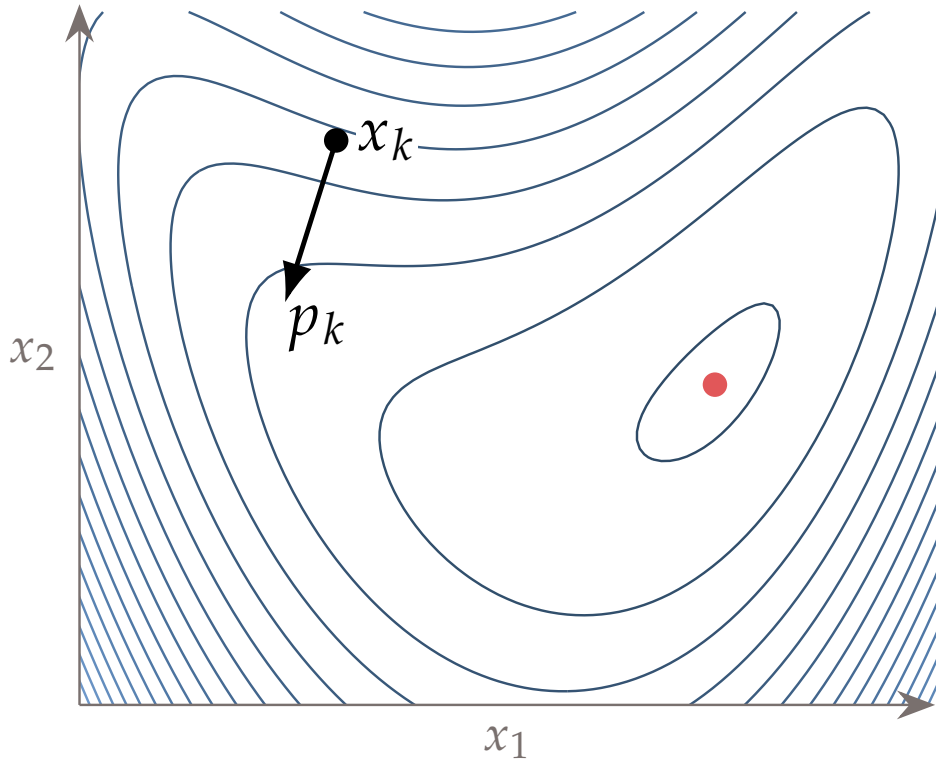


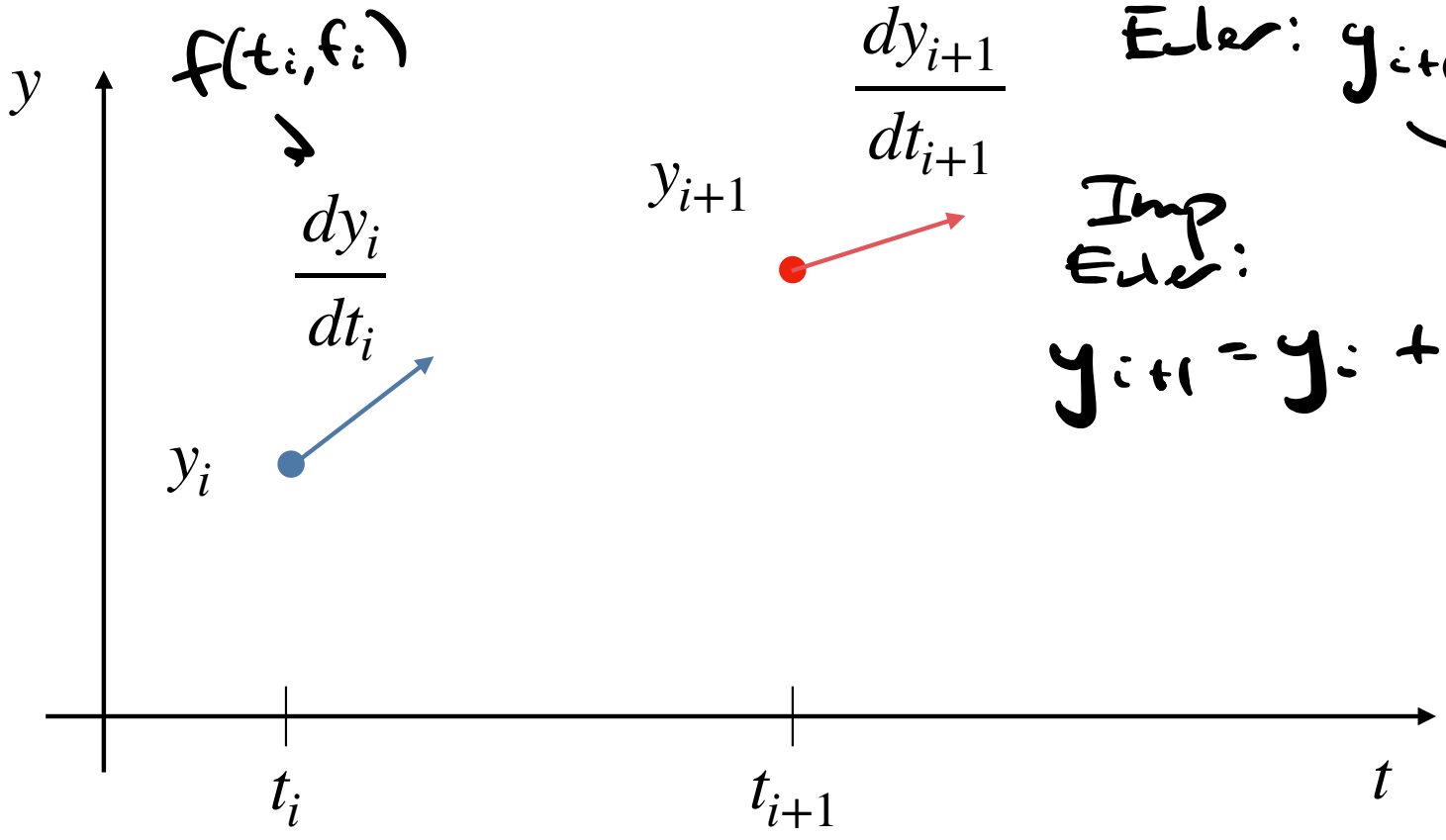
IVP in Python

try: odefun(t, y) = 0.1* y

scipy.integrate.solve_ivp

Improved Euler

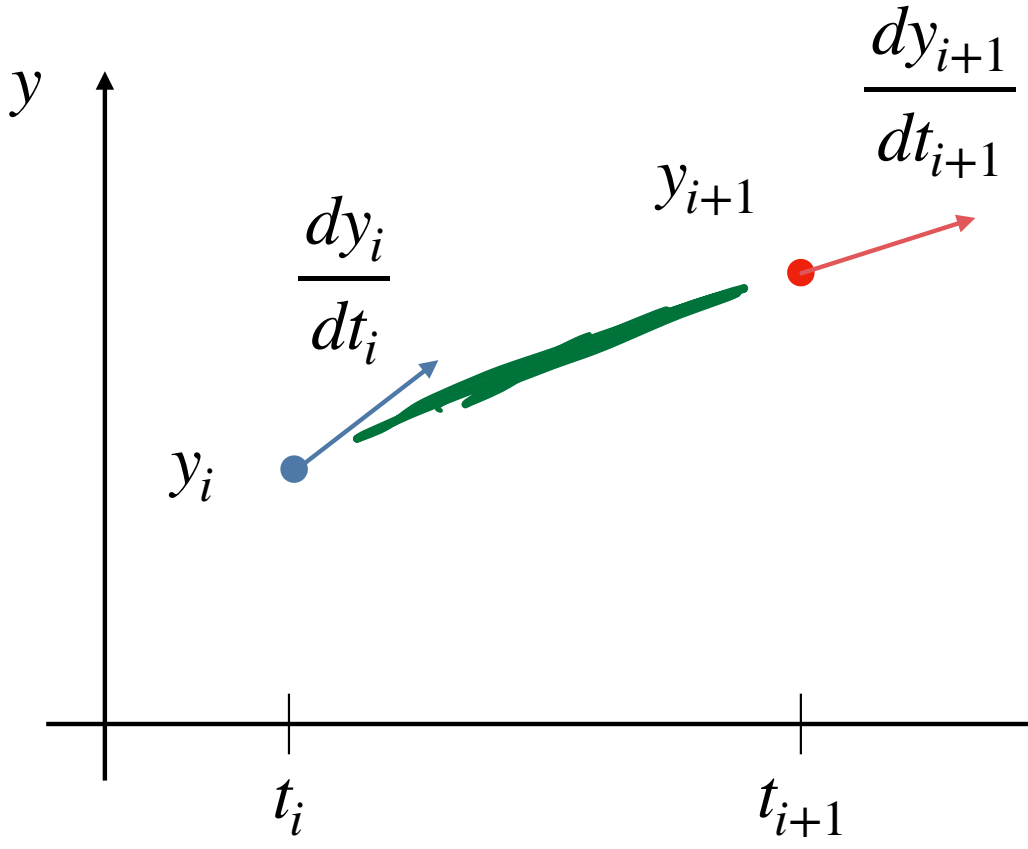
$f(t_{i+1}, y_{i+1})$ ↓ $f_i = f(t_i, y_i)$



Euler: $y_{i+1} = y_i + h f(t_i, y_i)$

Imp Euler: $y_{i+1} = y_i + h \frac{1}{2}(f_i + f_{i+1})$

Improved Euler



$$f_e = f(t_i, y_i)$$

$$f_r = f(t_i + h, y_i + h f_e)$$

$$y_{i+1} = y_i + \frac{h}{2}(f_e + f_r)$$

Runge Kutta Methods

RK4

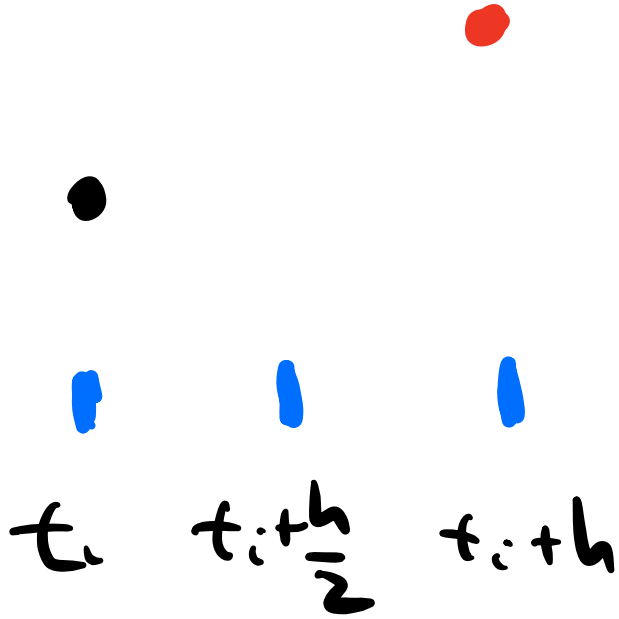
$$k_1 = f(t_i, y_i)$$

$$k_2 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_1\right)$$

$$k_3 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_2\right)$$

$$k_4 = f\left(t_i + h, y_i + hk_3\right)$$

$$y_{i+1} = y_i + \frac{h}{6}\left(k_2 + 2k_2 + 2k_3 + k_4\right)$$



Adaptive Time Stepping

Systems of ODEs

1 ODE → $\frac{dy}{dt} = f(t, y)$ y is a vector

$$\frac{dy_1}{dt} = f(t, y_1, y_2, y_3)$$

$$\frac{dy_2}{dt} = f(t, y_1, y_2, y_3)$$

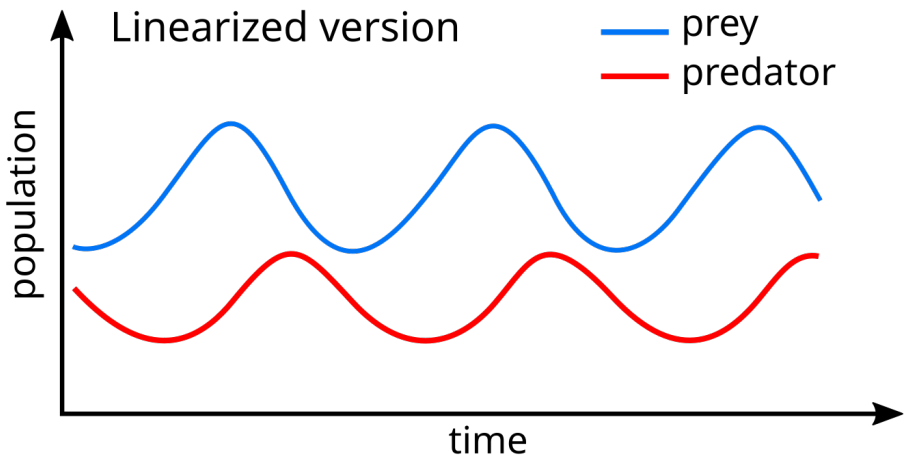
$$\frac{dy_3}{dt} = f(t, y_1, y_2, y_3)$$

3 initial conditions.

Example: Predator-Prey Model

$$\frac{du}{dt} = \alpha u - \beta uv$$

$$\frac{dv}{dt} = -\gamma v + \delta uv$$



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