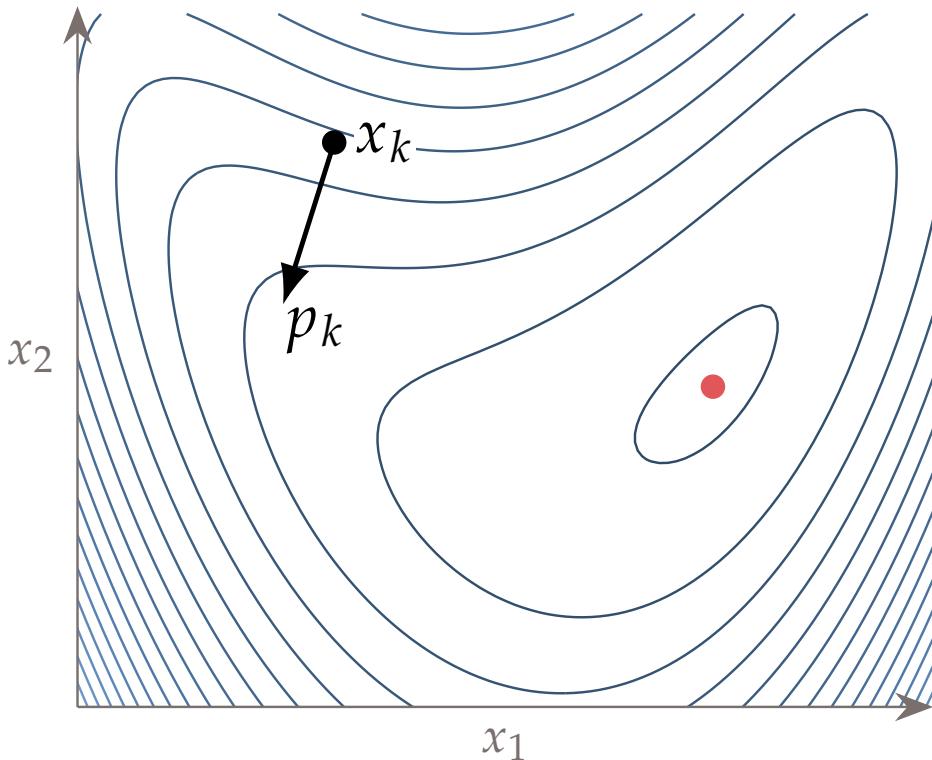


Differential Equations



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Where We Are

Algebra

Calculus

Linear
Algebra

Differential
Equations

1D root finding

integration

systems of eqns

differentiation

least squares

Where We Are

Algebra

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ID root finding

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systems of eqns

ND root finding

differentiation

least squares

optimization

eigenvalues

Motivation

$$\frac{dp}{dt} = rp$$

$$\frac{dT}{dt} = -k(T - T_{room})$$

$$-c\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin \theta = 0$$

First Order ODE

$$\frac{dy}{dt} = f(t, y)$$

want to solve for y

y : dependent

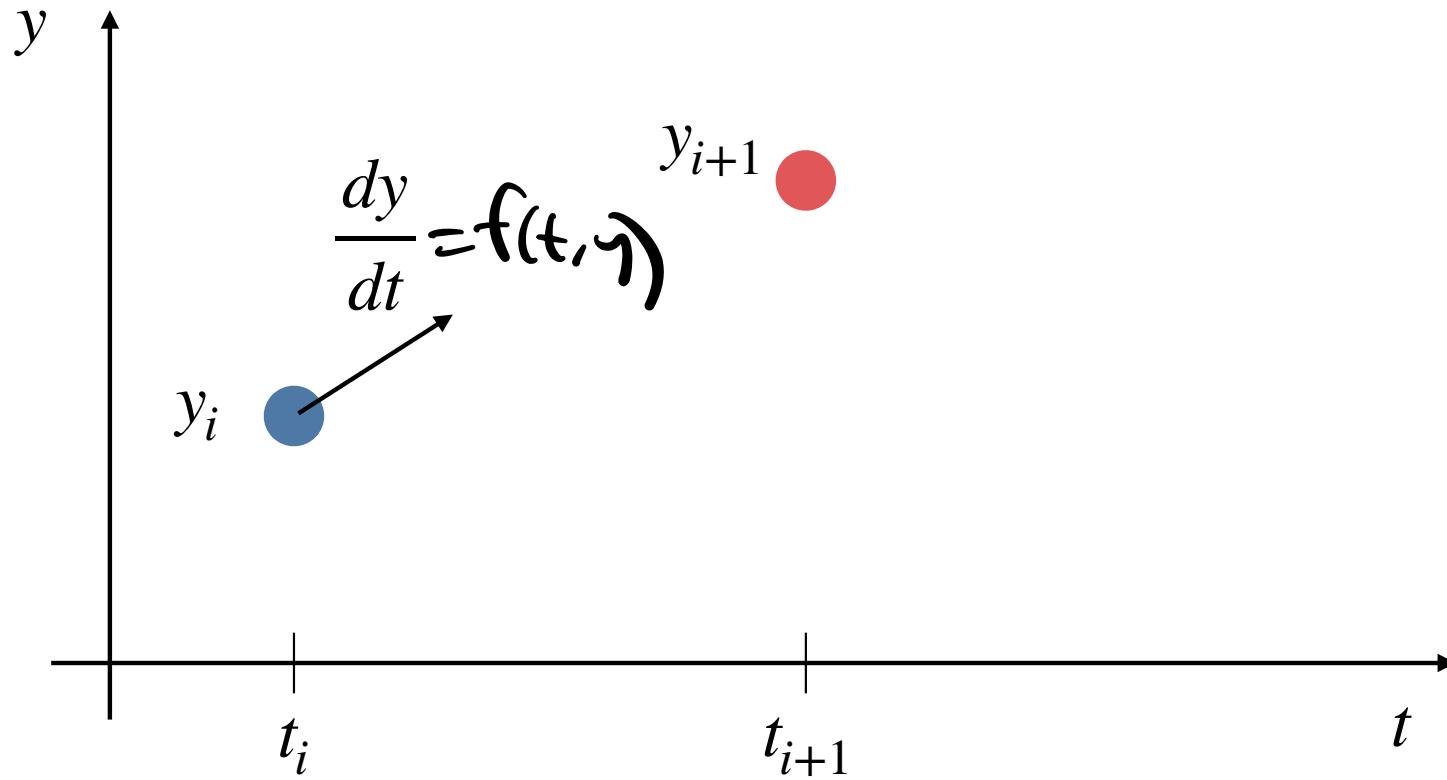
t : independent.

Initial Value Problem

$$\frac{dy}{dt} = f(t, y) \quad \text{want } y(t)$$

$$y(0) = y_0 \leftarrow \text{initial condition.}$$

How to do this numerically?



Euler's Method

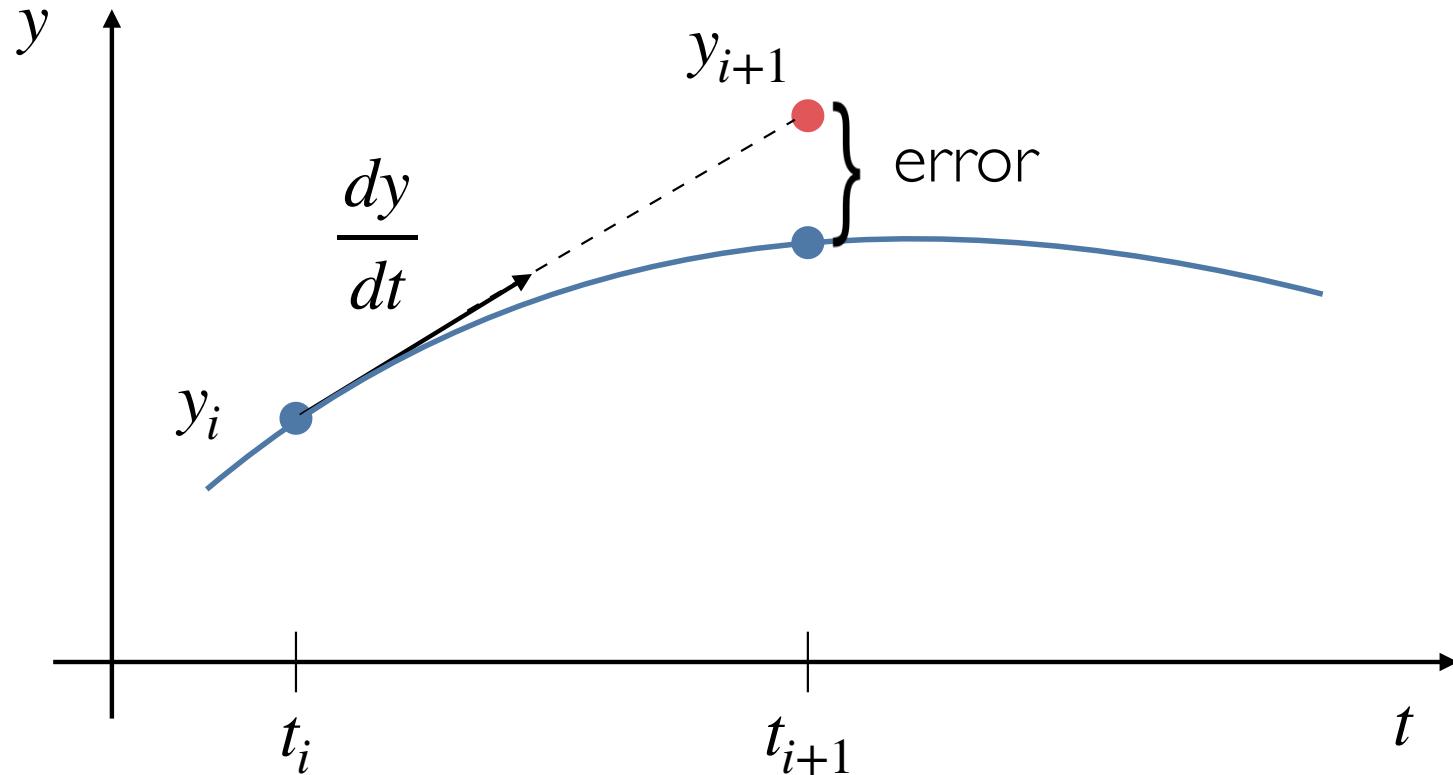
$$\frac{dy}{dt} = f(t, y) \quad \xrightarrow{\hspace{1cm}} \quad \frac{dy}{dt} \approx \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$$

$\underbrace{h}_{\text{blue}}$

$$f(t, y) = \frac{y_{i+1} - y_i}{h}$$

$$\Rightarrow y_{i+1} = y_i + h \cdot f(t_i, y_i)$$

Euler's Method



Write our own Euler ODE solver

`euler(f, y0, t0, tf, h)`

$$y_{i+1} = y_i + h \cdot f(t_i, y_i)$$

$f(t, y)$ - returns dy/dt

y_0 - starting point

t_0, tf - starting and final time

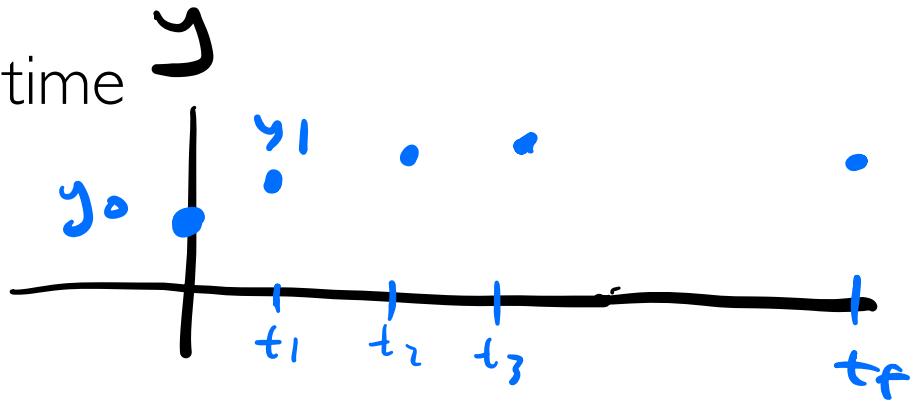
h - time step

$$y_0 = 1$$

$y(t)$

try: `odefun(t, y) = 0.1 * y`

$$\Rightarrow \frac{dy}{dt} = 0.1 y$$



Stability
